

Flux-induced Soft Supersymmetry Breaking in Chiral Type *IIB* Orientifolds with *D3/D7*-Branes

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Abstract

We discuss supersymmetry breaking via 3-form fluxes in chiral supersymmetric type *IIB* orientifold vacua with *D3*- and *D7*-branes. After a general discussion of possible choices of fluxes allowing for stabilizing of a part of the moduli, we determine the resulting effective action including all soft supersymmetry breaking terms. We also extend the computation of our previous work [1] concerning the matter field metrics arising from various open string sectors, in particular focusing on the 1/2 BPS *D3/D7*-brane configuration. Afterwards, the *F*-theory lift of our constructions is investigated.

1. Introduction

Type *I/II* superstring compactifications with *D*-branes are promising candidates to provide an effective 4-dimensional theory very similar to the Standard Model of particle physics. Concerning the question of how space-time supersymmetry is realized, at least two classes of models seem to be viable scenarios. First, space-time supersymmetry is broken by the *D*-brane configurations. In this case the supersymmetry breaking scale is expected to be near the string scale, and, as a consequence for solving the hierarchy problem, large internal dimensions are most likely required. Or, second, the open string sector from the *D*-branes preserves $N=1$ supersymmetry. Then it is very natural to consider supersymmetry breaking effects in the closed string bulk which generically transmute themselves by gravitational interactions into the observable open string Standard Model sector. From the effective field theory point of view one is dealing with a low-energy effective $N=1$ supergravity theory with soft supersymmetry breaking parameters, induced by the supersymmetry breaking effects in the bulk. It is this second class of models which we like to study in this paper in the context of chiral type *IIB* orientifolds with supersymmetric *D3*- and *D7*-brane configurations.

Non-trivial field strength fluxes for closed string p -form fields provide a natural mechanism for space-time supersymmetry breaking in the bulk as well as for stabilizing at least some of the moduli of the underlying compactification manifold. Many attempts in this direction start from type *IIB* superstring theory compactified on compact Calabi-Yau manifolds (CYM) with 3-form NS and R fluxes. An immediate consequence of allowing those non-vanishing fluxes is, that one in general needs to introduce extended objects in order to satisfy the Einstein equations of the low-energy supergravity description [2]. The string theoretical explanation to this is, that these fluxes imply positive or negative *D3*-brane charge, which has to be cancelled by adding objects producing the opposite of this charge. Generically, this then results in a warped form of the metric due to the back reaction of the branes. Typically, if the flux produces a positive *D3*-charge (so-called ISD fluxes), one may balance this charge by adding orientifold planes (of negative charge), anti-*D3*-branes, or (wrapped) *D7*-branes with or without 2-form fluxes.

Examples of such vacua are type *IIB* compactified on Calabi-Yau orientifolds (or toroidal orbifold/orientifolds) with *D3*-branes filling the uncompactified space-time. If the *D3* fills the uncompactified space-time, the low-energy effective action on the *D3*-brane is conformally invariant and described by $N=4$ supergravity. Only if the *D3*-brane sits at a singularity, like a conifold singularity of a CYM or an orbifold singularity of an orbifold compactification, the gauge theory on the *D3*-branes leads to an $N=1$ chiral spectrum. Hence, quite generically, apart from those special cases, the inclusion of fluxes leads to supersymmetry breaking from $N=4$ to non-chiral non-supersymmetric gauge theories. In order to obtain a chiral spectrum, one has to study more involved constructions. *E.g.* if in

addition to the $D3$ -branes one has $D7$ -branes with internal 2-form fluxes, one may obtain chiral non-supersymmetric theories after turning on 3-form fluxes. This setup, which we want to study in more detail in this article, is T -dual to type IIA orientifold models with intersecting $D6$ -branes [3,4,5], which were extensively discussed in the literature (for a review see [6])^{1,2}. Finally note that type IIB with $D3$ - and $D7$ -branes can also be described by F -theory, whose constant coupling limits describe type IIB compactified on CY orientifolds.

A type II compactification on a Calabi-Yau threefold X_6 leads to $h_{2,1}(X_6)$ complex structure and $h_{1,1}(X_6)$ Kähler moduli fields. Depending on how their related cohomology element behaves under the orientifold projection, some of the moduli fields are projected out. The remaining fields, together with the universal dilaton field, are the scalars of $N=1$ chiral multiplets. In the case of unbroken $N=1$ supersymmetry, and before turning on any fluxes, these moduli fields have flat potentials and thus remain undetermined. As we shall recall later, due to consistency, only so-called ISD-fluxes are allowed to be turned on at the level of lowest supergravity approximation [12]. After switching on such ISD-fluxes some or all of the complex structure moduli and the dilaton may be frozen at certain values as a result of flux quantization conditions. On the other hand, such fluxes do not generate a scalar potential for the Kähler moduli. Even if one allowed for IASD-fluxes, the situation would not be improved, since the flux-dependent superpotential only depends on the dilaton and complex structure moduli and a potential only for those moduli fields is generated after turning on those fluxes. Hence those moduli remain undetermined. Other mechanisms, like higher loop effects, world-sheet instanton effects or gaugino condensation were discussed to generate a potential for the Kähler moduli. Moreover, ISD fluxes will lead to a negative (or zero) cosmological constant. Even after taking into account effects to stabilize the Kähler moduli, the potential generically has a negative minimum. However, a small positive cosmological constant appears to be called for by recent experimental data. Hence, in addition to the effects mentioned above, which fix the Kähler moduli, other effects have to occur to generate a positive potential. As suggested in [13], this may be achieved by adding anti- $D3$ -branes. Our setup will be general enough to allow for these extra effects.

If one does not want to rely on the above mentioned higher loop effects, world-sheet instanton effects or gaugino condensation, the introduction of $D7$ -branes with 2-form fluxes in addition to the $D3$ -branes provides a natural way to fix also the Kähler moduli of the internal space. (Moduli stabilization in type IIB orientifolds with $D9$ and anti- $D9$ -branes

¹ Recent promising attempts to construct supersymmetric intersecting brane models can be found in [7,8,9].

² Type IIB models with $D9/D5$ -branes with magnetic \mathcal{F} -flux were also considered in *Refs.* [10,11].

and 3-form fluxes were considered in [14,15], whereas type *IIA* orientifolds with *D6*-branes and fluxes were discussed in [16].) Namely, the effective *D*-term scalar potential, which depends on the Kähler moduli, is due to the attractive (or repulsive) forces between the *D3*-branes and the *D7*-branes and also the orientifold planes, and in its (supersymmetric) minimum, most of the Kähler moduli are generically fixed. So putting together fluxes and different kinds of *D3/D7*-branes (or in type *IIA* intersecting *D6*-branes), both complex structure as well as Kähler moduli can be stabilized.

The main emphasis of this paper is on the computation of the soft supersymmetry breaking terms of the effective four-dimensional *D3/D7* chiral gauge theory action after turning on the 3-form *NS* and *R* fluxes in the bulk. Our derivation of the soft terms will be performed in the framework of the $D = 4$ effective $N=1$ supergravity action [17] where spontaneous supersymmetry breaking is due to the non-vanishing auxiliary *F*-term components of the moduli fields [18,19]. For the case of non-chiral orientifolds with only *D3*-branes, the corresponding soft terms were already computed in [20,21]. For intersecting *D6*-branes, a computation of soft terms has been undertaken in [22]. Specifically, we need the following two ingredients to compute the effective soft terms: First one has to determine the supersymmetry breaking *F*-terms, which originate from an effective bulk superpotential [23,24,25,26] due to the non-vanishing 3-form fluxes, as well as the effective action for the (closed string) moduli fields in Calabi-Yau orientifolds [27]. Second the knowledge of the $D = 4$ moduli dependent effective action of the open string gauge and matter fields on the *D3/D7* world volumes is required. Computing directly the relevant string scattering amplitudes of gauge, matter and moduli fields from intersecting *D6*-branes respective from *D9*-branes with 2-form fluxes, the open string effective action was recently obtained in [1]. Here we will compute, via mirror symmetry and also by direct string computations, the analogous effective action for the open string fields on the *D3/D7*-brane system. In particular, we derive the matter field metrics arising from various open string sectors, in particular focusing on the 1/2 BPS *D3/D7*-brane configuration. In order to deal with a specific set up, we consider an $\mathbf{Z}_2 \times \mathbf{Z}_2$ orientifold with *D3*- and *D7*-branes, where we will derive the complete set of flux induced soft supersymmetry breaking parameters.

The paper is organized as follows. In the next section we introduce type *IIB* orientifolds with *D3*- and *D7*-branes and 3-form fluxes, which are *T*-dual to type *IIA* orientifolds with intersecting *D6*-branes. In section 3 we extend the results from [1] concerning the open string effective action to the case of *D3*-branes together with *D7*-branes with 2-form fluxes. In chapter 4 we will then compute the effective action with the 3-form fluxes turned on, in particular the soft supersymmetry breaking terms. Finally, in section 5, the *F*-theory description of our models is provided.

2. Type *IIB* orientifolds with *D3*– and *D7*–branes and three–form fluxes

In this section, we shall construct type *IIB* orientifold models with *D3*– and *D7*–branes, where we will allow for internal open string 2-form f -fluxes turned on the various stacks of *D7*–branes. Chiral fermions will arise from open strings stretched between the *D3*–branes and the *D7*–branes, and also from open strings stretched between different stacks of *D7*–branes. These models can be obtained from type *IIA* orientifolds with intersecting *D6*–branes after performing T -duality transformations along three internal directions. The type *IIA* intersection angles between the *D6*–branes correspond to the different type *IIB* f -fluxes after the T -duality transformations. Specifically, we shall study the type *IIB* superstring compactified on a six–dimensional orbifold $X_6 = T^6/\Gamma$, with the discrete group $\Gamma = \mathbf{Z}_N$ or $\Gamma = \mathbf{Z}_N \times \mathbf{Z}_M$. Generically, this leads to an $N=2$ supersymmetric (closed string) spectrum in $D = 4$. In addition, to obtain an $N=1$ string spectrum one introduces an orientifold projection ΩI_n , with Ω describing a reversal of the orientation of the closed string world–sheet and I_n a reflection of n internal coordinates. For ΩI_n to represent a symmetry of the original theory, n has to be an even integer in type *IIB*. Moreover, in order that ΩI_n becomes also a \mathbf{Z}_2 –action in the fermionic sector, the action ΩI_n has to be supplemented by the operator $[(-1)^{F_L}]^{\lfloor \frac{n}{2} \rfloor}$. Here, $\lfloor \frac{n}{2} \rfloor$ represents the integer part of $n/2$. The operator $(-1)^{F_L}$ assigns a $+1$ eigenvalue to states from the $NSNS$ –sector and a -1 to states from the RR –sector. Generically, this projection produces orientifold fixed planes [$O(9-n)$ –planes], placed at the orbifold fixpoints of T^6/I_n . They have negative tension, which has to be balanced by introducing positive tension objects. Candidates for the latter may be collections of $D(9-n)$ –branes and/or non–vanishing three–form fluxes H_3 and C_3 . In order to obtain a consistent low–energy supergravity description, the above objects are subject to the supergravity equations of motion. Eventually, this puts restrictions on the possible choices of fluxes, to be discussed later. The orbifold group Γ mixes with the orientifold group ΩI_n . As a result, if the group Γ contains \mathbf{Z}_2 –elements θ , which leave one complex plane fixed, we obtain additional $O(9-|n-4|)$ – or $O(3+|n-2|)$ –planes from the element $\Omega I_n \theta$.

Eventually, we want to turn on vevs for the (untwisted) three–form fluxes H_3 and F_3 . This limits the possibilities for the choice of the orientifold projection ΩI_n . Since the NS 2–form B_2 is odd under Ω , we need to take $n \neq 0$ in order for non–vanishing 3–form flux components $H_{ijk} = \partial_{[i} B_{jk]}$ to survive the orientifold projection. In the case of ΩI_6 , all 20 real components H_{ijk} may receive non–vanishing vevs [28]. Here, I_6 is the \mathbf{Z}_2 –reflection of the three internal complex coordinates:

$$I_6 : z^i \longrightarrow -z^i \quad , \quad i = 1, 2, 3 . \quad (2.1)$$

2.1. $\mathbf{Z}_2 \times \mathbf{Z}_2$ orientifold with D3- and D7-branes

As a concrete example, we concentrate on the $\Gamma = \mathbf{Z}_2 \times \mathbf{Z}_2$ toroidal orbifold (without discrete torsion), with the two group generators θ, ω acting in the following way

$$\begin{aligned}\theta &: (z^1, z^2, z^3) \longrightarrow (-z^1, -z^2, z^3) , \\ \omega &: (z^1, z^2, z^3) \longrightarrow (z^1, -z^2, -z^3)\end{aligned}\tag{2.2}$$

on the three internal complex coordinates z^i , $i = 1, 2, 3$. Furthermore, the six-torus T^6 is assumed to be a direct product of three two-tori $T^{2,j}$, *i.e.* $T^6 = \otimes_{j=1} T^{2,j}$. The manifold $(T^2)^3/(\mathbf{Z}_2 \times \mathbf{Z}_2)$ has the Hodge numbers $h_{(1,1)} = 3$ and $h_{(2,1)} = 51$. Hence, there are three Kähler moduli \mathcal{T}^j ,

$$\mathcal{T}^j = a^j + i R_1^j R_2^j \sin \alpha^j \quad , \quad j = 1, 2, 3 \quad ,\tag{2.3}$$

describing the size of the three tori $T^{2,j}$, with the metric:

$$g_j = \begin{pmatrix} (R_1^j)^2 & R_1^j R_2^j \cos \alpha^j \\ R_1^j R_2^j \cos \alpha^j & (R_2^j)^2 \end{pmatrix} .\tag{2.4}$$

Here, the axions a^j stem from reducing the RR 4-form on the 4-cycles $T^{2,k} \times T^{2,l}$, *i.e.* $a^j = \int_{T^{2,k} \times T^{2,l}} C_4$. Besides the three complex structure moduli

$$\mathcal{U}^j = \frac{R_2^j}{R_1^j} e^{i\alpha^j} \quad , \quad j = 1, 2, 3 \quad ,\tag{2.5}$$

there are 48 additional ones, which represent blowing up moduli. The latter refer to the 3×16 fixpoints resulting from the orbifold group elements θ, ω and $\theta\omega$. These orbifold singularities are of real codimension 4. The respective modulus corresponds to a $C_2 \times C_1$ cycle. The C_1 refers to a \mathbf{P}^1 , which is collapsed at the orbifold singularity and the C_2 denotes the torus, which is fixed under the respective orbifold group. Hence we have 48 collapsed 3-cycles $C_2 \times C_1$ of type (2, 1) and (1, 2). A detailed discussion of the massless spectrum of this model will be given in the next section. There is one subtlety concerning the correct field definitions. The moduli fields $\mathcal{T}^j, \mathcal{U}^j$ we get from the geometry (we call them the moduli in the string basis) are not scalars of chiral multiplets, so we need to make a basis transformation to obtain the physical fields suitable for a field theory calculation. We will now introduce the moduli S, T^j, U^j in the field theory basis. For the complex structure moduli, we do not need a redefinition in type *IIB*, $\mathcal{U}^i = U^i$. The imaginary part of the Kähler moduli is given by the coupling of the gauge fields on a $D7$ -brane $g_{D7,j}^{-2}$, which is wrapped on the tori $T^{2,k}$ and $T^{2,l}$. So

$$T^j = a^j + i \frac{e^{-\phi_4}}{2\pi\alpha'^{1/2}} \sqrt{\frac{\text{Im } \mathcal{T}^k \text{Im } \mathcal{T}^l}{\text{Im } \mathcal{T}^j}} .\tag{2.6}$$

The imaginary part of the dilaton S is given by the gauge coupling on the $D3$ -brane g_{D3}^{-2} :

$$S = C_0 + i \frac{e^{-\phi_4}}{2\pi} \frac{\alpha'^{3/2}}{\sqrt{\text{Im } \mathcal{T}^1 \text{Im } \mathcal{T}^2 \text{Im } \mathcal{T}^3}} . \quad (2.7)$$

As described at the beginning, the orbifold group (2.2) implies the additional orientifold actions $\Omega I_6 \theta$, $\Omega I_6 \omega$ and $\Omega I_6 \theta \omega$. The latter essentially correspond to the generators ΩI_2^3 , ΩI_2^1 and ΩI_2^2 , respectively. The generators I_2^j reflect only one complex coordinate z^j :

$$I_2^j : z^j \longrightarrow -z^j . \quad (2.8)$$

The orientifold action (2.8) implies 64 $O3$ -planes ΩI_6 and $4 \times 3 = 12$ $O7$ -planes ΩI_2^k , $k = 1, 2, 3$. The latter are sitting at the four fixed-points of each $T^{2,j}/I_2^j$. These orientifold planes produce a negative C_4 and C_8 -form potential, which has to be cancelled. These potentials may be balanced by placing $D3$ -branes and $D7$ -branes on top of the orientifold planes. To obtain a chiral spectrum, we may introduce (magnetic) two-form fluxes $F^j dx^j \wedge dy^j$ on the internal part of $D7$ -brane world volume. Together with the internal NS B -field³ b^j we combine the complete 2-form flux into $\mathcal{F} = \sum_{j=1}^3 \mathcal{F}^j := \sum_{j=1}^3 (b^j + 2\pi\alpha' F^j) dx^j \wedge dy^j$. The latter gives rise to the total internal antisymmetric background

$$\begin{pmatrix} 0 & f^j \\ -f^j & 0 \end{pmatrix} , \quad f^j = \frac{1}{(2\pi)^2} \int_{T^{2,j}} \mathcal{F}^j , \quad (2.9)$$

w.r.t. the j -th internal plane. The 2-form fluxes \mathcal{F}^j have to obey the quantization rule:

$$m^j \frac{1}{(2\pi)^2 \alpha'} \int_{T^{2,j}} \mathcal{F}^j = n^j , \quad n \in \mathbf{Z} , \quad (2.10)$$

i.e. $f^j = \alpha' \frac{n^j}{m^j}$. We obtain non-vanishing instanton numbers

$$m^j m^k \int_{T^{2,j} \times T^{2,k}} \mathcal{F} \wedge \mathcal{F} = (2\pi)^4 \alpha'^2 n^j n^k \quad (2.11)$$

on the world-volume of a $D7$ -brane, which is wrapped around the 4-cycle $T^{2,j} \times T^{2,k}$ with the wrapping numbers m^j . Hence, through the CS -coupling $T_7 C_4 \wedge \mathcal{F} \wedge \mathcal{F}$, a $D7$ -brane may also induce contributions to the 4-form potential. Note, that a $D3$ -brane may be described by a $D7$ -brane with $f^j \rightarrow \infty$. To cancel the tadpoles arising from the Ramond

³ Note, that b^j has to be quantized due to the orientifold projection Ω to the values $b^j = 0$ or $b^j = \frac{1}{2}$ [29].

forms C_4 and C_8 , we introduce N_{D3} (space-time filling) $D3$ -branes and K stacks of $D7$ -branes with internal fluxes. More concretely, K^i stacks of N_a^i $D7$ -branes with internal 2-form fluxes $\mathcal{F}^j, \mathcal{F}^k$ and wrapping numbers m_a^j, m_a^k w.r.t. the 4-cycle $T^{2,j} \times T^{2,k}$. The cancellation condition for the tadpole arising from the RR 4-form C_4 is

$$N_{D3} + \frac{2}{(2\pi)^4 \alpha'^2} \sum_{\substack{(i,j,k) \\ =(1,2,3)}} \sum_{a=1}^{K^i} N_a^i m_a^j m_a^k \int_{T^{2,j} \times T^{2,k}} \mathcal{F} \wedge \mathcal{F} = 32 , \quad (2.12)$$

i.e. according to Eq. (2.11):

$$N_{D3} + 2 \sum_{\substack{(i,j,k) \\ =(1,2,3)}} \sum_{a=1}^{K^i} N_a^i n_a^j n_a^k = 32 . \quad (2.13)$$

Furthermore, the cancellation conditions for the 8-form tadpoles yield:

$$\begin{aligned} 2 \sum_{a=1}^{K^3} N_a^3 m_a^1 m_a^2 &= -32 , \\ 2 \sum_{a=1}^{K^2} N_a^2 m_a^1 m_a^3 &= -32 , \\ 2 \sum_{a=1}^{K^1} N_a^1 m_a^2 m_a^3 &= -32 . \end{aligned} \quad (2.14)$$

The extra factor of 2 in front of the sums over the $D7$ -brane stacks accounts for additional mirror branes. For each $D7$ -brane with wrapping numbers (m^i, m^j) , we also have to take into account its mirror $(-m^i, -m^j)$ in order to cancel induced RR 6-form charges. The r.h.s. of Eqs. (2.12) and (2.14) accounts for the contributions of the $O3$ - and $O7$ -planes, respectively. An $O3$ -plane contributes $-\frac{1}{4}$ of a $D3$ -brane charge T_3 . In the covering space the 64 $O3$ -planes are doubled, thus contributing $2 \times 64 \times (-\frac{1}{4}) = -32$ on the l.h.s. of (2.13). On the other hand, in $D7$ -brane charge T_7 units, an $O7$ -plane contributes $4T_7$. Hence, in the covering space, four $O7$ -planes contribute $2 \times 4 \times 4 = 32$ on the l.h.s. of (2.14).

A $D3$ -brane placed in the uncompactified $D = 4$ space-time produces the contribution

$$V_{D3} = T_3 e^{-\phi_4} \frac{\alpha'^{3/2}}{\sqrt{T_2^1 T_2^2 T_2^3}} \quad (2.15)$$

to the total scalar potential V . Here $T_p = (2\pi)^{-p} \alpha'^{-\frac{1}{2} - \frac{p}{2}}$ is the Dp -brane tension [30] and $\phi_4 = \phi_{10} - \frac{1}{2} \ln [\text{Im}(\mathcal{T}^1) \text{Im}(\mathcal{T}^2) \text{Im}(\mathcal{T}^3) / \alpha'^3]$ the dilaton field in $D = 4$. Furthermore, a

$D7$ -brane, wrapped around the 4-cycle $T^{2,j} \times T^{2,k}$ with wrapping numbers m^j, m^k and internal gauge fluxes f^k, f^l gives rise to the potential

$$V_{D7_j} = -T_7 (2\pi)^4 \alpha'^{3/2} e^{-\phi_4} m^k m^l \left| 1 + i \frac{f^k}{T_2^k} \right| \left| 1 + i \frac{f^l}{T_2^l} \right| \sqrt{\frac{T_2^k T_2^l}{T_2^j}} . \quad (2.16)$$

In order that the $D7$ -branes preserve some supersymmetry, their internal 2-form fluxes f^i, f^j must obey the supersymmetry condition [31]:

$$\frac{f^i}{\text{Im} \mathcal{T}^i} = - \frac{f^j}{\text{Im} \mathcal{T}^j} . \quad (2.17)$$

In that case, the potential (2.16) simplifies:

$$V_{D7_j} = -T_7 (2\pi)^4 \alpha'^{3/2} e^{-\phi_4} m^k m^l \left(1 - \frac{f^k f^l}{T_2^k T_2^l} \right) \sqrt{\frac{T_2^k T_2^l}{T_2^j}} . \quad (2.18)$$

Hence, the presence of N_{D3} space-time filling $D3$ -branes and various stacks of $D7$ -branes produces a positive potential⁴

$$V_{D3/D7} = N_{D3} V_{D3} + 2 \sum_{j=1}^3 \sum_{a=1}^{K^j} N_a^j V_{D7_j} . \quad (2.19)$$

Furthermore, a negative potential is generated by the presence of the 64 $O3$ - and 12 $O7_j$ -orientifold planes:

$$V_{O3/O7} = 2e^{-\phi_4} \alpha'^{3/2} \left\{ -64T'_3 \frac{1}{\sqrt{T_2^1 T_2^2 T_2^3}} - 4T'_7 (2\pi)^4 \left(\sqrt{\frac{T_2^2 T_2^3}{T_2^1}} + \sqrt{\frac{T_2^1 T_2^3}{T_2^2}} + \sqrt{\frac{T_2^1 T_2^2}{T_2^3}} \right) \right\} . \quad (2.20)$$

Here, the orientifold tension for Op -planes is given by $T'_p = 2^{p-5} T_p$ [30]. The extra factor of 2 is due to the covering space. In the case of supersymmetric $D7$ -branes, *i.e.* (2.17) holding for each brane, we have

$$V_{D3/D7} + V_{O3/O7} = 0 , \quad (2.21)$$

provided the tadpole conditions (2.12) and (2.14) are fulfilled.

The simplest solution to the equations (2.12) and (2.14) is represented by the following example: We take 32 space-time filling $D3$ -branes and place 8 $D7$ -branes on top of each of the 12 $O7$ -planes. This leads to a non-chiral spectrum and the 96 $D7$ -branes give rise

⁴ The extra factor of two in front of the $D7$ -brane sum accounts for the mirror branes.

to the gauge group $SO(8)^{12}$ [32]. A more involved example, which leads to a chiral N=1 spectrum, can be found:

Stack	Gauge group	(m^1, n^1)	(m^2, n^2)	(m^3, n^3)	N_a
1	$U(2)$	—	—	—	4 $D3$
2	$USp(8)$	—	(1, 0)	(−1, 0)	8 $D7$
3	$USp(8)$	(1, 0)	—	(−1, 0)	8 $D7$
4	$U(3) \times U(1)$	(1, 1)	(−2, 1)	—	8 $D7$
5	$USp(4)$	—	(2, 1)	(−1, 1)	4 $D7$
6	$U(1)$	(2, 1)	—	(−2, 1)	2 $D7$

Table 1: Chiral $D3/D7$ brane configuration: wrapping numbers m^j , internal flux numbers n^j and amount of supersymmetry preserved.

The supersymmetry condition (2.17) may be fulfilled for each stack, provided the three Kähler moduli obey $\text{Im}T^1 = \text{Im}T^3 = T$ and $\text{Im}T^2 = \frac{1}{2}T$. Hence, in that case the NS -tadpoles are cancelled as well, *i.e.* (2.21) holds. In addition, two of the three Kähler moduli T^j are fixed as a result of demanding a chiral supersymmetric vacuum solution. The full configuration preserves N=1 supersymmetry in $D = 4$. After performing T -dualities in the three x -directions of the three tori $T^{2,j}$, the above configuration leads to the supersymmetric intersecting $D6$ -brane model, introduced in [5].

Together with the complex dilaton field

$$S = \tau = C_0 + ie^{-\phi_{10}} , \quad (2.22)$$

we have $h_{(1,1)} + h_{(2,1)} + 1 = 55$ chiral N=1 multiplets from the closed string sector (bulk): seven from the untwisted sector and 48 from the twisted sector. Additional chiral multiplets arise from the open string sector. The $D3$ -branes have six real transversal positions ϕ^i , which combine into 3 complex scalars. Furthermore the $D7$ -branes which are wrapped around a 4-cycle give rise to one complex scalar describing the transversal movement and two Wilson line moduli. Moreover, there are moduli accounting for bundles on the $D7$ -branes. In addition, there are moduli from the twisted (open string) sector, describing scalar matter fields. We shall come to a detailed discussion of the spectrum in section 3.

2.2. Turning on the 3-form fluxes

Let us now give non-vanishing vevs to some of the flux components H_{ijk} and F_{ijk} , with $F_3 = dC_2$, $H_3 = dB_2$. A thorough discussion of 3-form fluxes on the torus $(T^2)^3$ is presented in appendix A. On the torus T^6 , we would have 20+20 independent internal components for H_{ijk} and F_{ijk} . In addition, all these components remain inert under the orientifold projection $\Omega(-1)^{F_L} I_6$. However, under ΩI_2 , only a subset of 12 of the 20 flux

components survive [28]. Hence, after taking into account all three I_2 -projections I_2^j , we are left with only eight flux components:

$$H_{135} , H_{136} , H_{145} , H_{146} , H_{235} , H_{236} , H_{245} , H_{246} . \quad (2.23)$$

A similar analysis applies to the RR 3-form flux components $F_{ijk} = \partial_{[i} C_{jk]}$, whose 2-form potential C_2 is odd under $(-1)^{F_L}$, *i.e.* $\Omega(-1)^{F_L} C_2 = -C_2$. With similar arguments as before, the components

$$F_{135} , F_{136} , F_{145} , F_{146} , F_{235} , F_{236} , F_{245} , F_{246} \quad (2.24)$$

survive the orientifold projections $\Omega(-1)^{F_L} I_2^j$, $j = 1, 2, 3$. So far, the effect of the orbifold action Γ has not yet been taken into account and the components (2.23) and (2.24) represent the set of fluxes, invariant under both I_2^j , $j = 1, 2, 3$ and I_6 . Note, that the fluxes (2.23) and (2.24) are automatically invariant under the orbifold group (2.2) as well and no further components are lost. In fact, these 8 real flux components correspond to a linear combination of the $2h_{2,1} + 2h_{3,0} = 8$ primitive elements of the cohomology $H^3(X_6, \mathbf{C})$ (*cf.* the next subsection). To summarize, we shall consider the type IIB orbifold $(T^2)^3/(\mathbf{Z}_2 \times \mathbf{Z}_2)$ supplemented with the additional orientifold $\Omega(-1)^{F_L} I_6$ action and obtain an $N=1$ (non-chiral) spectrum in the closed string sector. After performing three T -duality actions in all six internal coordinates, this model becomes the non-chiral $D9/D5$ -orientifold model of [33].

The fluxes in (2.23) and (2.24) have to obey the quantization rules $\frac{1}{(2\pi)^2 \alpha'} \int_{C_3} F_3 \in \mathbf{Z}$ and $\frac{1}{(2\pi)^2 \alpha'} \int_{C_3} H_3 \in \mathbf{Z}$, with $F_3, H_3 \in H^3(X_6, \mathbf{Z})$. We shall make some comments in the following. It has been pointed out in *Ref.* [34], that there are subtleties for toroidal orientifolds due to additional 3-cycles, which are not present in the covering space T^6 . If some integers are odd, additional discrete flux has to be turned on in order to meet the quantization rule for those 3-cycles. We may bypass these problems in our concrete $\mathbf{Z}_2 \times \mathbf{Z}_2$ orientifold, if we choose the quantization numbers to be multiples of 8 and do not allow for discrete flux at the orientifold planes [14,15]. Note, that in addition to the untwisted flux components H_{ijk} and F_{ijk} there may be also $NSNS$ - and RR -flux components from the twisted sector. We do not consider them here. It is assumed, that their quantization rules freeze the blowing up moduli at the orbifold singularities. The extra 48 3-cycles, which are collapsed at those singularities, do not give rise to extra quantization conditions for the untwisted flux components. In fact, the flux integrals over those give zero [35].

The two 3-forms F_3, H_3 are organized in the $SL(2, \mathbf{Z})_S$ covariant field:

$$G_3 = F_3 - S H_3 . \quad (2.25)$$

After giving a vev to the field G_3 , the Chern–Simons term⁵

$$\mathcal{S}_{CS} = \frac{1}{2} \frac{1}{(2\pi)^7 \alpha'^4} \int \frac{C_4 \wedge G_3 \wedge \overline{G}_3}{S - \overline{S}} . \quad (2.26)$$

of the ten–dimensional effective type *IIB* action gives rise to an additional tadpole for the *RR* four–form C_4 (in units of T_3):

$$N_{flux} = \frac{1}{(2\pi)^4 \alpha'^2} \int_{X_6} H_3 \wedge F_3 . \quad (2.27)$$

Hence in the presence of 3–form fluxes, the tadpole condition (2.12) is modified to:

$$N_{flux} + N_{D3} + \frac{2}{(2\pi)^4 \alpha'^2} \sum_{\substack{(i,j,k) \\ =(1,2,3)}} \sum_{a=1}^{K^i} N_a^i m_a^j m_a^k \int_{T^{2,j} \times T^{2,k}} \mathcal{F} \wedge \mathcal{F} = 32 . \quad (2.28)$$

The *CP* even analog of (2.27) originates from the piece $\frac{-1}{2 \cdot 3!} \frac{1}{(2\pi)^7 \alpha'^4} \int d^{10}x \sqrt{-g_{10}} |G_3|^2$ of the $D = 10$ type *IIB* action and leads to the potential term in $D = 4$:

$$\frac{1}{2 (2\pi)^7 \alpha'^4} \int_{X_6} d^6y G_3 \wedge \star_6 \overline{G}_3 . \quad (2.29)$$

According to [12,36], the latter may be split into a purely topological term V_{top} , independent of the moduli fields, and a second term V_{flux} , relevant for the F –term contribution to the scalar potential. After the decomposition $G_3 = G^{ISD} + G^{IASD}$, with $\star_6 G^{ISD} = +i G^{ISD}$ and $\star_6 G^{IASD} = -i G^{IASD}$, one obtains [12,36,37]:

$$\begin{aligned} V_{flux} &= \frac{1}{(2\pi)^7 \alpha'^4} \int_{X_6} G^{IASD} \wedge \star_6 \overline{G}^{IASD} , \\ V_{top} &= -e^{-\phi_{10}} T_3 N_{flux} . \end{aligned} \quad (2.30)$$

Hence, the total contributions to the scalar potential are:

$$\begin{aligned} V &= V_D + V_F , \\ V_D &= V_{D3/D7} + V_{O3/O7} + V_{top} , \\ V_F &= V_{flux} . \end{aligned} \quad (2.31)$$

The piece V_D represents D –term contributions to the scalar potential due to Fayet–Iliopolous terms. See *Ref.* [38] for further details. Only the last term corresponds to

⁵ Throughout this section, we work in the string–frame, *i.e.* with the Einstein term $\frac{1}{(2\pi)^7 \alpha'^4} \int d^{10}x \sqrt{-g_{10}} e^{-2\phi_{10}} R$.

an F -term. For the case, that the conditions (2.28) and (2.14) are met, Ramond tadpole contributions are absent. If in addition, (2.17) is met, *i.e.* only supersymmetric 2-form fluxes on the $D7$ -brane world-volume are considered, the first three terms add up to zero: $V_{D3/D7} + V_{O3/O7} + V_{top} = 0$, *i.e.* $V_D = 0$. Let us remark, that this condition may generically also fix some of the Kähler moduli \mathcal{T}^j . In the following, we shall assume, that $V_D = 0$ and study only the F -term contribution $V_F = V_{flux}$ to the scalar potential V . The potential V_F , displayed in Eq. (2.31), originates from the closed string sector only. It is derived from the superpotential⁶ [24]:

$$\widehat{W} = \frac{\lambda}{(2\pi)^2 \alpha'} \int_{X_6} G_3 \wedge \Omega . \quad (2.32)$$

2.3. The 3-form fluxes on $(T^2)^3/\mathbf{Z}_2 \times \mathbf{Z}_2$

We will now work out the explicit form of the 3-form flux G_3 in terms of the closed string moduli U^i and S . Expressed in real coordinates, we take the following basis of 3-form fluxes allowed on $(T^2)^3/\mathbf{Z}_2 \times \mathbf{Z}_2$:

$$\begin{aligned} \alpha_0 &= dx^1 \wedge dx^2 \wedge dx^3 & \beta^0 &= dy^1 \wedge dy^2 \wedge dy^3 \\ \alpha_1 &= dy^1 \wedge dx^2 \wedge dx^3 & \beta^1 &= -dx^1 \wedge dy^2 \wedge dy^3 \\ \alpha_2 &= dx^1 \wedge dy^2 \wedge dx^3 & \beta^2 &= -dy^1 \wedge dx^2 \wedge dy^3 \\ \alpha_3 &= dx^1 \wedge dx^2 \wedge dy^3 & \beta^3 &= -dy^1 \wedge dy^2 \wedge dx^3 \end{aligned} \quad (2.33)$$

This basis has the property $\int_{X_6} \alpha_i \wedge \beta^j = \delta_i^j$. The above fluxes all fulfill the primitivity condition $\alpha_i \wedge J = \beta^i \wedge J = 0$, with J the Kähler form. These are at the same time exactly the fluxes that are allowed in a setup with three stacks of $D7$ branes, one stack not wrapping T_3^2 , one not wrapping T_2^2 , and one not wrapping T_1^2 , see appendix 1.

Expressed in this basis, the G_3 -flux (2.25) takes the following form:

$$\frac{1}{(2\pi)^2 \alpha'} G_3 = \sum_{i=0}^3 \{ (a^i - S c^i) \alpha_i + (b_i - S d_i) \beta^i \}. \quad (2.34)$$

A basis of $H^3 = H^{(3,0)} \oplus H^{(2,1)} \oplus H^{(1,2)} \oplus H^{(0,3)}$, corresponding to the fluxes (2.33) is

$$\begin{aligned} dz^1 \wedge dz^2 \wedge dz^3, \quad d\bar{z}^1 \wedge dz^2 \wedge dz^3, \quad dz^1 \wedge d\bar{z}^2 \wedge dz^3, \quad dz^1 \wedge dz^2 \wedge d\bar{z}^3, \\ dz^1 \wedge d\bar{z}^2 \wedge d\bar{z}^3, \quad d\bar{z}^1 \wedge dz^2 \wedge d\bar{z}^3, \quad d\bar{z}^1 \wedge d\bar{z}^2 \wedge dz^3, \quad d\bar{z}^1 \wedge d\bar{z}^2 \wedge d\bar{z}^3, \end{aligned} \quad (2.35)$$

⁶ The factor of λ serves to obtain the correct mass dimension of 3 for the superpotential, *i.e.* $\lambda \propto \kappa_4^{-3}$

where $dz^i = dx^i + U^i dy^i$. We now want to express this basis through our real basis $\{\alpha_i, \beta^j\}$ and the complex structure moduli of $T^2 \times T^2 \times T^2$:

$$\begin{aligned}
\omega_{A0} &= \alpha_0 + \sum_{i=1}^3 \alpha_i \overline{U^i} - \beta_1 \overline{U^2 U^3} - \beta_2 \overline{U^1 U^3} - \beta_3 \overline{U^1 U^2} + \beta_0 \overline{U^1 U^2 U^3} \\
\omega_{A1} &= \alpha_0 + \alpha_1 \overline{U^1} + \alpha_2 \overline{U^2} + \alpha_3 \overline{U^3} - \beta_1 \overline{U^2 U^3} - \beta_2 \overline{U^1 U^3} - \beta_3 \overline{U^1 U^2} + \beta_0 \overline{U^1 U^2 U^3} \\
\omega_{A2} &= \alpha_0 + \alpha_1 \overline{U^1} + \alpha_2 \overline{U^2} + \alpha_3 \overline{U^3} - \beta_1 \overline{U^2 U^3} - \beta_2 \overline{U^1 U^3} - \beta_3 \overline{U^1 U^2} + \beta_0 \overline{U^1 U^2 U^3} \\
\omega_{A3} &= \alpha_0 + \alpha_1 \overline{U^1} + \alpha_2 \overline{U^2} + \alpha_3 \overline{U^3} - \beta_1 \overline{U^2 U^3} - \beta_2 \overline{U^1 U^3} - \beta_3 \overline{U^1 U^2} + \beta_0 \overline{U^1 U^2 U^3} \\
\omega_{B0} &= \alpha_0 + \sum_{i=1}^3 \alpha_i U^i - \beta_1 U^2 U^3 - \beta_2 U^1 U^3 - \beta_3 U^1 U^2 + \beta_0 U^1 U^2 U^3 \\
\omega_{B1} &= \alpha_0 + \alpha_1 U^1 + \alpha_2 \overline{U^2} + \alpha_3 \overline{U^3} - \beta_1 \overline{U^2 U^3} - \beta_2 U^1 \overline{U^3} - \beta_3 U^1 \overline{U^2} + \beta_0 U^1 \overline{U^2 U^3} \\
\omega_{B2} &= \alpha_0 + \alpha_1 \overline{U^1} + \alpha_2 U^2 + \alpha_3 \overline{U^3} - \beta_1 U^2 \overline{U^3} - \beta_2 \overline{U^1 U^3} - \beta_3 \overline{U^1 U^2} + \beta_0 \overline{U^1 U^2 U^3} \\
\omega_{B3} &= \alpha_0 + \alpha_1 \overline{U^1} + \alpha_2 \overline{U^2} + \alpha_3 U^3 - \beta_1 \overline{U^2 U^3} - \beta_2 \overline{U^1 U^3} - \beta_3 \overline{U^1 U^2} + \beta_0 \overline{U^1 U^2 U^3}
\end{aligned} \tag{2.36}$$

ω_{B0} obviously corresponds to the $(3, 0)$ -part of the flux and the Calabi-Yau 3-form Ω can be normalized to equal ω_{B0} . ω_{A1} , ω_{A2} and ω_{A3} are the $(2, 1)$ -components of the flux, ω_{B1} , ω_{B2} and ω_{B3} the $(1, 2)$ -components of the flux, and ω_{A0} corresponds to the $(0, 3)$ -part, *i.e.* $\overline{\Omega}$.

Note that this basis is not normalized to one. It fulfills

$$\begin{aligned}
\int \omega_{Ai} \wedge \omega_{Bi} &= \prod_{i=1}^3 (U^i - \overline{U^i}), \quad i = 0, \dots, 3 \\
\int \omega_{Aj} \wedge \omega_{Bk} &= 0, \quad j \neq k.
\end{aligned} \tag{2.37}$$

Expressed in this basis, the G_3 -flux takes the following form:

$$\frac{1}{(2\pi)^2 \alpha'} G_3 = \sum_{i=0}^3 (A^i \omega_{Ai} + B^i \omega_{Bi}). \tag{2.38}$$

By comparing the coefficients of G_3 expressed in the real basis and in the complex basis and solving for the $\{A^i, B^i\}$, we can express the $\{A^i, B^i\}$ as a function of $\{a^i, c^i, b_i, d_i\}$ and the moduli fields S, U^i . By setting the respective coefficients to zero, we obtain equations for the respective flux parts. This gives us constraints on the $\{a^i, c^i, b_i, d_i\}$. What has to be taken into account as well is the fact that the $\{a^i, c^i, b_i, d_i\}$ must be integer numbers. This requirement can only be fulfilled for specific choices of the U^i and of S , *i.e.* it fixes the moduli.

Expressed with the coefficients of the real basis, we find N_{flux} , given in (2.27), to be

$$N_{\text{flux}} = \sum_{i=0}^3 c^i b_i - \sum_{i=0}^3 a^i d_i.$$

We want to find the corresponding expression in complex language. We find

$$\begin{aligned} N_{\text{flux}} &= \frac{1}{(2\pi)^4 (\alpha')^2} \frac{1}{(S - \bar{S})} \int \bar{G}_3 \wedge G_3 \\ &= -\frac{\prod_{i=1}^3 (U^i - \bar{U}^i)}{(S - \bar{S})} \sum_{i=0}^3 (|A^i|^2 - |B^i|^2), \end{aligned} \quad (2.39)$$

which is quite a nice expression. And it immediately teaches us something about the behaviour of the different fluxes: The fluxes obeying the ISD-condition, i.e. those having all $B_i = 0$, have $N_{\text{flux}} > 0$, whereas the IASD-fluxes, i.e. those with all $A_i = 0$ have $N_{\text{flux}} < 0$.

(i) *The supersymmetric case: (2,1)-flux*

It is common knowledge that turning on only the (2,1)-part of the 3-form flux does not break supersymmetry. Such a flux fulfills the ISD condition and from eq. (2.30), we know that $V_{\text{flux}} = 0$.

We obtain the necessary equations by setting the coefficients of the (0,3)-, (1,2)-, and (3,0)-flux to zero, i.e. $A_0 = B_0 = B_1 = B_2 = B_3 = 0$. This corresponds to the following equations:

$$\begin{aligned} 0 &= U^1 U^2 U^3 (a^0 - S c_0) - \sum_{i \neq j \neq k} (a^i - S c^i) U^j U^k - (b_0 - S d_0) - \sum_{i=1}^3 (b_i - S d_i) U^i \\ 0 &= \bar{U}^1 U^2 U^3 (a^0 - S c_0) - \{(a^1 - S c^1) U^2 U^3 + (a^2 - S c^2) \bar{U}^1 U^3 + (a^3 - S c^3) \bar{U}^1 U^2\} - \\ &\quad - (b_0 - S d_0) - \{(b_1 - S d_1) \bar{U}^1 + (b_2 - S d_2) U^2 + (b_3 - S d_3) U^3\} \\ 0 &= \bar{U}^1 \bar{U}^2 \bar{U}^3 (a^0 - S c_0) - \sum_{i \neq j \neq k} (a^i - S c^i) \bar{U}^j \bar{U}^k - (b_0 - S d_0) - \sum_{i=1}^3 (b_i - S d_i) \bar{U}^i \\ 0 &= U^1 \bar{U}^2 U^3 (a^0 - S c_0) - \{(a^1 - S c^1) \bar{U}^2 U^3 + (a^2 - S c^2) U^1 U^3 + (a^3 - S c^3) U^1 \bar{U}^2\} - \\ &\quad - (b_0 - S d_0) - \{(b_1 - S d_1) U^1 + (b_2 - S d_2) \bar{U}^2 + (b_3 - S d_3) U^3\} \\ 0 &= U^1 U^2 \bar{U}^3 (a^0 - S c_0) - \{(a^1 - S c^1) U^2 \bar{U}^3 + (a^2 - S c^2) U^1 \bar{U}^3 + (a^3 - S c^3) U^1 U^2\} - \\ &\quad - (b_0 - S d_0) - \{(b_1 - S d_1) U^1 + (b_2 - S d_2) U^2 + (b_3 - S d_3) \bar{U}^3\}, \end{aligned} \quad (2.40)$$

with $i \neq j \neq k$. There is another, more elegant way of obtaining the equations for the $(2, 1)$ -flux: G_3 not having a $(0, 3)$ -part is equivalent to requiring that

$$\int G_3 \wedge \Omega = 0,$$

which yields the first of the above equations. G_3 not having a $(3, 0)$ -part is equivalent to requiring that

$$\int G_3 \wedge \bar{\Omega} = 0,$$

which yields the second of the above equations, and G_3 not having a $(1, 2)$ -part is equivalent to requiring that

$$\int G_3 \wedge \omega_{A1} = \int G_3 \wedge \omega_{A2} = \int G_3 \wedge \omega_{A3} = 0,$$

which gives us the remaining three equations. (We remember that ω_{A1} , ω_{A2} , ω_{A3} are the basis of the $(2, 1)$ -flux.) The integration picks out the allowed flux components and we obtain the same equations as above in a different way.

There is another equivalent way to obtain the equations for the $(2, 1)$ -flux. We know that for the flux to be supersymmetric, we must impose the conditions

$$\begin{aligned} \widehat{W} &= \frac{\lambda}{(2\pi)^2 \alpha'} \int G_3 \wedge \Omega = 0, \\ D_S \widehat{W} &= \partial_S \widehat{W} + \kappa_4^2 \widehat{W} \partial_S \widehat{K} = 0, \\ D_{U^i} \widehat{W} &= \partial_{U^i} \widehat{W} + \kappa_4^2 \widehat{W} \partial_{U^i} \widehat{K} = 0, \end{aligned} \tag{2.41}$$

where D_M is the Kähler covariant derivative and the Kähler potential \widehat{K} is given in the next section. We want to check, whether these conditions are really equivalent to the ones we found before. The first condition obviously corresponds to our first equation. After multiplication of the second condition with $-(S - \bar{S})$, we find it to correspond to $\int \bar{G}_3 \wedge \Omega = 0$, which is the complex conjugate of the second of our equations. After multiplying the third of the conditions with $-(U^i - \bar{U}^i)$, we find it to correspond to $\int G_3 \wedge \omega_{Ai}$, so we have complete equivalence between our equations and the conditions above.

Now we can solve for the $\{a^i, c^i, b_i, d_i\}$ and impose the constraint that they have to be integer numbers. These constraints cannot be solved in full generality, i.e. for arbitrary moduli and flux coefficients. By fixing some of the moduli and/or flux coefficients, it is possible to obtain special solutions. Here, we choose to fix the moduli to $U^1 = U^2 = U^3 = S = i$.

One possible solution for the $(2, 1)$ -flux we get is

$$\begin{aligned}
\frac{1}{(2\pi)^2\alpha'} G_{21} = & [-d_0 + i(d_1 + d_2 + d_3)] \alpha_0 + [-d_1 - i(-b_2 - b_3 + d_0)] \alpha_1 \\
& + (-d_2 - ib_2) \alpha_2 + (-d_3 - ib_3) \alpha_3 + (-d_1 - d_2 - d_3 - id_0) \beta^0 \\
& + (-b_2 - b_3 + d_0 - id_1) \beta^1 + (b_2 - id_2) \beta^2 + (b_3 - id_3) \beta^3,
\end{aligned} \tag{2.42}$$

where $b_2, b_3, d_0, d_1, d_2, d_3$ can be any integer number. As reported in section 2, we can avoid possible complications with flux quantization, if we take our flux coefficients to be multiples of 8. This can be achieved by simply taking $b_2, b_3, d_0, d_1, d_2, d_3$ to be multiples of 8.

Expressed in the complex basis (2.36), the solution takes the form

$$\begin{aligned}
\frac{1}{(2\pi)^2\alpha'} G_{21} = & \frac{1}{2} [-b_2 - b_3 + i(d_2 + d_3)] \omega_{A1} + \frac{1}{2} [b_2 - d_0 + i(d_1 + d_3)] \omega_{A2} + \\
& + \frac{1}{2} [b_3 - d_0 + i(d_1 + d_2)] \omega_{A3}.
\end{aligned} \tag{2.43}$$

For N_{flux} we find

$$\begin{aligned}
N_{\text{flux}} = & 4 (|A^1|^2 + |A^2|^2 + |A^3|^2) \\
= & 2 \left(\sum_{i=0}^3 d_i^2 + d_1 d_2 + d_1 d_3 + d_2 d_3 + b_2^2 + b_3^2 + b_2 b_3 - b_2 d_0 - b_3 d_0 \right).
\end{aligned} \tag{2.44}$$

If we require N_{flux} to have a certain value, this places quite stringent constraints on our choice for the coefficients. The smallest possible N_{flux} for our solution, the coefficients being multiples of 8, is $N_{\text{flux}} = 128$. To achieve this, we have several possibilities. We can for example set either of the d_i or b_i to ± 8 , and all the other coefficients to zero. For $d_0 = 8$ for example, all other coefficients being zero, this would amount to

$$\frac{1}{(2\pi)^2\alpha'} G_3 = 8 (-\alpha_0 - i\alpha_1 - i\beta^0 + \beta^1).$$

Another possible solution would be $b_2 = 8$, $b_3 = -8$, or the other way round. This would result in

$$\frac{1}{(2\pi)^2\alpha'} G_3 = 8 (-i\alpha_2 + i\alpha_3 + \beta^2 - \beta^3).$$

(ii) $(0, 3)$ -flux

This flux meets the ISD-condition as well, therefore $V_{\text{flux}} = 0$.

To obtain the $(0, 3)$ -part of the flux, we must set $A_1 = A_2 = A_3 = B_0 = B_1 = B_2 = B_3 = 0$ or equivalently require that $\int G_3 \wedge \bar{\Omega} = \int G_3 \wedge \omega_{A1} = \int G_3 \wedge \omega_{A2} = \int G_3 \wedge \omega_{A3} = \int G_3 \wedge \omega_{B1} = \int G_3 \wedge \omega_{B2} = \int G_3 \wedge \omega_{B3} = 0$. This results in the following seven equations:

$$\begin{aligned}
0 &= U^1 \bar{U}^2 \bar{U}^3 (a^0 - S c_0) - \{(a^1 - S c^1) \bar{U}^2 \bar{U}^3 + (a^2 - S c^2) \bar{U}^1 \bar{U}^3 + (a^3 - S c^3) U^1 \bar{U}^2\} - \\
&\quad - (b_0 - S d_0) - \{(b_1 - S d_1) U^1 + (b_2 - S d_2) \bar{U}^2 + (b_3 - S d_3) \bar{U}^3\} \\
0 &= \bar{U}^1 U^2 \bar{U}^3 (a^0 - S c_0) - \{(a^1 - S c^1) U^2 \bar{U}^3 + (a^2 - S c^2) \bar{U}^1 \bar{U}^3 + (a^3 - S c^3) \bar{U}^1 U^2\} - \\
&\quad - (b_0 - S d_0) - \{(b_1 - S d_1) \bar{U}^1 + (b_2 - S d_2) U^2 + (b_3 - S d_3) \bar{U}^3\} \\
0 &= \bar{U}^1 \bar{U}^2 U^3 (a^0 - S c_0) - \{(a^1 - S c^1) \bar{U}^2 U^3 + (a^2 - S c^2) \bar{U}^1 U^3 + (a^3 - S c^3) \bar{U}^1 \bar{U}^2\} - \\
&\quad - (b_0 - S d_0) - \{(b_1 - S d_1) \bar{U}^1 + (b_2 - S d_2) \bar{U}^2 + (b_3 - S d_3) U^3\} \\
0 &= \bar{U}^1 \bar{U}^2 \bar{U}^3 (a^0 - S c_0) - \sum_{i \neq j \neq k} (a^i - S c^i) \bar{U}^j \bar{U}^k - (b_0 - \tau d_0) - \sum_{i=1}^3 (b_i - S d_i) \bar{U}^i \\
0 &= \bar{U}^1 U^2 U^3 (a^0 - S c_0) - \{(a^1 - S c^1) U^2 U^3 + (a^2 - S c^2) \bar{U}^1 U^3 + (a^3 - S c^3) \bar{U}^1 U^2\} - \\
&\quad - (b_0 - S d_0) - \{(b_1 - S d_1) \bar{U}^1 + (b_2 - S d_2) U^2 + (b_3 - S d_3) U^3\} \\
0 &= U^1 \bar{U}^2 U^3 (a^0 - S c_0) - \{(a^1 - S c^1) \bar{U}^2 U^3 + (a^2 - S c^2) U^1 U^3 + (a^3 - S c^3) U^1 \bar{U}^2\} - \\
&\quad - (b_0 - S d_0) - \{(b_1 - S d_1) U^1 + (b_2 - S d_2) \bar{U}^2 + (b_3 - S d_3) U^3\} \\
0 &= U^1 U^2 \bar{U}^3 (a^0 - S c_0) - \{(a^1 - S c^1) U^2 \bar{U}^3 + (a^2 - S c^2) U^1 \bar{U}^3 + (a^3 - S c^3) U^1 U^2\} - \\
&\quad - (b_0 - S d_0) - \{(b_1 - S d_1) U^1 + (b_2 - S d_2) U^2 + (b_3 - S d_3) \bar{U}^3\},
\end{aligned} \tag{2.45}$$

with $i \neq j \neq k$. Now we solve again for the $\{a^i, c^i, b_i, d_i\}$, and after fixing the moduli $U^1 = U^2 = U^3 = S = i$ get a solution for the $(0, 3)$ -flux:

$$\begin{aligned}
\frac{1}{(2\pi)^2 \alpha'} G_{03} &= (d_0 + i d_3) \alpha_0 + (d_3 - i d_0) \alpha_1 \\
&\quad + (d_3 - i d_0) \alpha_2 + (d_3 - i d_0) \alpha_3 - (d_3 - i d_0) \beta^0 \\
&\quad + (d_0 + i d_3) \beta^1 + (d_0 + i d_3) \beta^2 + (d_0 + i d_3) \beta^3.
\end{aligned} \tag{2.46}$$

We see, that we now have much stronger constraints than in the $(2, 1)$ -case, which is no surprise, as we also have more equations to fulfill. When we could choose any integer numbers for $b_2, b_3, d_0, d_1, d_2, d_3$ in the $(2, 1)$ -case, we can now only choose the values for d_0 and d_3 , which we again take to be multiples of 8. Expressed in the complex basis (2.36), the solution takes the form

$$\frac{1}{(2\pi)^2 \alpha'} G_{03} = (d_0 + i d_3) \omega_{A0}. \tag{2.47}$$

For the $(0, 3)$ -flux, we find

$$N_{\text{flux}} = 4 |A_0|^2 = 4 (d_3^2 + d_0^2).$$

Here, the smallest possible N_{flux} is 256. There are four possible solutions for $N_{\text{flux}} = 256$, namely $d_3 = \pm 8$, $d_0 = 0$ and $d_0 = \pm 8$, $d_3 = 0$. For $d_3 = 8$, $d_0 = 0$, this results in

$$\frac{1}{(2\pi)^2 \alpha'} G_3 = 8 (i\alpha_0 + \alpha_1 + \alpha_2 + \alpha_3 - \beta^0 + i\beta^1 + i\beta^2 + i\beta^3).$$

(iii) $(1, 2)$ -flux

This flux is IASD, and therefore not consistent with the supergravity equations of motion.

Here, we require $A_0 = A_1 = A_2 = A_3 = B_0 = 0$ or equivalently $\int G_3 \wedge \Omega = \int G_3 \wedge \bar{\Omega} = \int G_3 \wedge \omega_{B1} = \int G_3 \wedge \omega_{B2} = \int G_3 \wedge \omega_{B3} = 0$. We will refrain from writing down the equations for this case, which are just a different combination of the equations we have seen before. A solution for $U^1 = U^2 = U^3 = S = i$ is

$$\begin{aligned} \frac{1}{(2\pi)^2 \alpha'} G_{12} = & [d_0 + i(d_1 + d_2 + d_3)] \alpha_0 + [d_1 - i(b_2 + b_3 + d_0)] \alpha_1 \\ & + (d_2 + ib_2) \alpha_2 + (d_3 + ib_3) \alpha_3 + [d_1 + d_2 + d_3 - id_0] \beta^0 \\ & + (-b_2 - b_3 - d_0 - id_1) \beta^1 + (b_2 - id_2) \beta^2 + (b_3 - id_3) \beta^3, \end{aligned} \quad (2.48)$$

which differs from the $(2, 1)$ -case only by signs. Expressed in the complex basis (2.36), the solution takes the form

$$\begin{aligned} \frac{1}{(2\pi)^2 \alpha'} G_{12} = & \frac{1}{2} [-b_2 - b_3 + i(d_2 + d_3)] \omega_{B1} + \frac{1}{2} [b_2 + d_0 + i(d_1 + d_3)] \omega_{B2} + \\ & + \frac{1}{2} [b_3 + d_0 + i(d_1 + d_2)] \omega_{B3}. \end{aligned} \quad (2.49)$$

For $(1, 2)$ -flux, we find

$$\begin{aligned} N_{\text{flux}} = & -4 (|B^1|^2 + |B^2|^2 + |B^3|^2) \\ = & -2 \left(\sum_{i=0}^3 d_i^2 + d_1 d_2 + d_1 d_3 + d_2 d_3 + b_2 b_3 + b_2 d_0 + b_3 d_0 + b_2^2 + b_3^2 \right). \end{aligned} \quad (2.50)$$

As remarked before, for IASD-fluxes, N_{flux} is negative. The largest possible value for this solution is $N_{\text{flux}} = -128$, which is achieved whenever we choose one of the b_i or d_i equal to ± 8 and the others all equal to zero, or whenever we choose two coefficients to have the absolute value of 8, but with differing signs, and all other coefficients equal to zero.

(iv) (3, 0)-flux

This is an IASD-flux as well, again not consistent with the supergravity equations of motion.

To obtain the (3, 0)-part of the flux, we must set $A_0 = A_1 = A_2 = A_3 = B_1 = B_2 = B_3 = 0$ or equivalently require that $\int G_3 \wedge \Omega = \int G_3 \wedge \omega_{A1} = \int G_3 \wedge \omega_{A2} = \int G_3 \wedge \omega_{A3} = \int G_3 \wedge \omega_{B1} = \int G_3 \wedge \omega_{B2} = \int G_3 \wedge \omega_{B3} = 0$. As a solution for $U^1 = U^2 = U^3 = S = i$ we get

$$\begin{aligned} \frac{1}{(2\pi)^2 \alpha'} G_{30} &= (d_0 + id_3) \alpha_0 + (d_3 + id_0) \alpha_1 \\ &+ (d_3 + id_0) \alpha_2 + (d_3 + id_0) \alpha_3 - (d_3 + id_0) \beta^0 \\ &+ (d_0 - id_3) \beta^1 + (d_0 - id_3) \beta^2 + (d_0 - id_3) \beta^3, \end{aligned} \quad (2.51)$$

which is the complex conjugate of our solution for the (0, 3)-flux. Expressed in the complex basis (2.36), the solution takes the form

$$\frac{1}{(2\pi)^2 \alpha'} G_{30} = (d_0 - id_3) \omega_{B0}. \quad (2.52)$$

In the case of (3, 0)-flux,

$$N_{\text{flux}} = -4|B_0|^2 = -4(d_0^2 + d_3^2).$$

Here, the largest possible value for this solution is $N_{\text{flux}} = -256$, which we get for the same choices of coefficients as in the (0, 3)-case.

2.4. Examples of type IIB orientifold models with non-vanishing fluxes

In order to satisfy the supergravity equations of motion, the flux combination (2.25) has to obey the imaginary self-duality condition $\star_6 G = +i G$ [12]. This condition ensures the existence of a solution for the metric and 4-form. It also has crucial implications for the possible supersymmetry breaking scenarios and for the choice of four-dimensional space-time. From (2.30) we immediately see, that in this case no F -term contributions to the scalar potential V are produced, *i.e.* $V_F = V_{flux} = 0$. Only the D -term contribution $V_{top} \sim N_{flux}$ is non-vanishing. As already mentioned in section 2, this term enters V_D , given in (2.31).

Note, as discussed in section 2, the quantization conditions for the fluxes force the coefficients to be multiples of eight. This implies $|N_{flux}| \geq 64$. For ISD-fluxes we have $N_{flux} > 0$. According to Eq. (2.28), consistent solutions, *i.e.* vanishing Ramond tadpoles, may be possible without $D3$ -branes, *i.e.* $N_{D3} = 0$. In that case, the $O3$ -planes and/or non-trivial internal gauge bundles on the $D7$ -brane world-volume may balance the positive $D3$ -brane charge introduced by the ISD-fluxes. This corresponds to the initial step, proposed

in [13], to fix the dilaton and complex structure moduli only by ISD-fluxes. With that amount of flux, *i.e.* $N_{flux} = 128$ and the stacks of $D7$ -branes, displayed in Table 2, the conditions (2.28) and (2.14) are met with $N_{D3} = 0$.

Stack	Gauge group	(m^1, n^1)	(m^2, n^2)	(m^3, n^3)	N_a
1, 2, 3, 4	$SO(8)^4$	(1, 0)	(−1, 0)	—	4×4 $D7$
5, 6, 7, 8	$SO(8)^4$	(1, 0)	—	(−1, 0)	4×4 $D7$
9, 10, 11, 12	$SO(14)^4$	—	(1, 0)	(−1, 0)	4×7 $D7$
13	$U(12)$	—	(1, 2)	(1, −2)	12 $D7$

Table 2: $D7$ -brane configuration allowing for $N_{flux} = 128$: wrapping numbers m^j , internal flux numbers n^j and amount of supersymmetry preserved.

The first twelve stacks (and their mirrors) of $D7$ -branes are located at the twelve $O7$ -planes. Besides, the first eight stacks cancel tadpoles from the C_8 -form locally. In order for the last stack to preserve the supersymmetry condition (2.17), we need to fix the Kähler moduli: $\mathcal{T}^2 = \mathcal{T}^3$. An example leading to $N_{flux} = 128$ has been presented in subsection 2.3(i).⁷

As a final step, it has been suggested in [13] to include anti- $D3$ branes, *i.e.* $N_{D3} < 0$. This requires additional positive flux contributions in order for *Eqs.* (2.28) and (2.14) now to be satisfied. However, in that case, cancellation of RR -tadpoles does no longer imply the absence of the $NSNS$ -tadpoles, since anti- $D3$ -branes contribute to (2.28) with a negative sign in contrast to (2.15). Hence, a positive D -term potential remains due to the uncanceled $NSNS$ -tadpoles. In fact, for anti- $D3$ -branes with ISD-fluxes, soft-supersymmetry breaking terms are generated and supersymmetry is broken [20]. This may be a favorable scenario (*cf.* [13]), since the soft-supersymmetry breaking terms fix the positions of the anti- $D3$ -branes.

According to (2.30), only fluxes G_3 of IASD-type give rise to an F -term contribution V_F to the scalar potential V in $D = 4$ space-time dimensions. The latter is positive semidefinite. On the other hand, $N_{flux} < 0$ for IASD-fluxes and generically, $D3$ -branes are needed to fulfill (2.28). The supergravity equations of motions are inconsistent for such configurations [12]. This fact is related to uncanceled $NSNS$ -tadpoles and a positive scalar potential V .

⁷ Note that $N=1$ supersymmetry is only mutually preserved within the first three stacks of $D7$ -branes. However, the fourth stack, being itself $N=2$ supersymmetric, is non-supersymmetric w.r.t. the other three. An alternative, completely supersymmetric model with fluxes is provided by keeping the first three stacks of $D7$ -branes with $N_1 = N_2 = N_3 = 16$ and $N_{flux} = 32$. Hence, for this model fluxes through twisted cycles are needed [14,15]

ISD-fluxes do not lead to a potential for the $D3$ -matter fields C_i^3 [20,21]. Hence in that case, those moduli remain undetermined and one has to find other mechanism to stabilize them. Contrarily, $IASD$ -fluxes do not fix the matter fields C_i^3 of anti- $D3$ -branes.

3. The effective action of toroidal type IIB orbifold/orientifolds with $D3$ -and $D7$ -branes

It is well known, that any $N=1$ supergravity action in four space-time dimensions is encoded by three functions, namely the Kähler potential K , the superpotential W , and the gauge kinetic function f [17]. When such an effective action arises from a higher dimensional string theory, these three functions usually depend (non-trivially) on moduli fields describing the background of the present string model. In string compactifications with D -branes, one has to distinguish between closed string moduli from the bulk, and open string moduli related to the D -branes. The tree-level effective action describing the couplings of open and closed string moduli up to second derivative order has been determined for toroidal orbifold/orientifold models with $D3/D7$ -branes or $D5/D9$ -branes and 2-form fluxes by computing string scattering amplitudes [1]. The low-energy action for only the closed string bulk moduli has been discussed for type IIB Calabi-Yau orientifolds in [21,27].

There are several types of moduli fields in a type IIB orientifold compactification with D -branes. The closed string moduli fields arise from dimensional reduction of the bosonic part $(\phi, g_{MN}, b_{MN}, C_0, C_2, C_4)$ of the $N=2$ supergravity multiplet in $D = 10$ after imposing the orientifold and orbifold action. In the following, let us concentrate on type IIB toroidal orbifolds/orientifolds with $D3$ - and $D7$ -branes. Hence, the spectrum has to be invariant under both the orientifold action $\Omega(-1)^{F_L} I_6$ and the orbifold group Γ . Before applying the orbifold twist Γ , the untwisted sector constitutes the states invariant under $\Omega(-1)^{F_L} I_6$: $\phi, g_{ij}, b_{\mu i}, C_0, C_{\mu i}, C_{ijkl}, C_{\mu\nu ij}, C_{\mu\nu\rho\sigma}$. The fields $\phi \equiv \phi_{10}$ and C_0 constitute the universal dilaton field (2.22). The field $C_{\mu\nu\rho\sigma}$ gives rise to a tadpole to be cancelled by the mechanism described in the previous section. Imposing on all the states encountered above the orbifold action⁸ Γ gives rise to the closed string untwisted sector. In addition there are twisted moduli comprising the twisted RR -tensors.

⁸ The orbifold group $\Gamma = \mathbf{Z}_2 \times \mathbf{Z}_2$, with the generators given in (2.2), leads to the following bosonic fields from the untwisted sector: $\phi, C_0, C_{\mu\nu\rho\sigma}, C_{\mu\nu j\bar{j}}, C_{i\bar{i} j\bar{j}}$, and the internal metric g has to have the block-diagonal structure $g = \otimes_{j=1}^3 g_j$, with g_j given in Eq. (2.4). The latter gives rise to 9 metric moduli, out of which the three complex structure moduli \mathcal{U}^j (2.5) and three real Kähler moduli $\text{Im}\mathcal{T}^j$ (2.3) are built. The latter are complexified with the three axions $a^j = \int_{T^{2,k} \times T^{2,l}} C_{k\bar{k}l\bar{l}}, (j, k, l) = (1, 2, 3)$. In $D = 4$, these scalars are dual to the anti-symmetric 2-tensors $\int_{T^{2,j}} C_{\mu\nu j\bar{j}}$ and are eliminated as a result of imposing self-duality on the self-dual 4-form C_4 . As already reported in the previous section, the twisted sector contains 48 additional complex structure moduli.

Let us now come to the open string moduli fields. The massless untwisted moduli fields originate from the $D = 10$ gauge field A_M reduced on the various D -branes. The orientifold projection Ω just determines the allowed Chan–Paton gauge degrees of freedom at the open string endpoints. For a stack of space–time filling $D3$ -branes, we obtain 6 real scalars ϕ^i , $i = 4, \dots, 9$ in the adjoint of the gauge group of the respective stack. These scalars describe the transversal movement of the $D3$ -branes, *i.e.* essentially the location of the $D3$ -branes on the six–dimensional compactification manifold. They may be combined into the three complex fields $C_i^3 = \phi^{2i+2} + \mathcal{U}^i \phi^{2i+3}$, $i = 1, 2, 3$. Furthermore, for a stack of $D7_3$ -branes, which is wrapped around the 4-cycle $T^{2,1} \times T^{2,2}$, we obtain the four Wilson lines A_i , $i = 4, 5, 6, 7$ and two transversal coordinates ϕ^8, ϕ^9 in the adjoint representation. The latter describe the position of the $D7$ -brane on the 2-torus $T^{2,3}$. Again, these six real fields may be combined into three complex fields $C_i^{7_3}$, $i = 1, 2, 3$. After taking into account the other two 4-cycles⁹, on which other stacks of $D7$ -branes may be wrapped, in total, we obtain the complex fields $C_i^{7_j}$, $i, j = 1, 2, 3$. All the open string fields described so far, give rise to complex scalars of the untwisted sector with at least N=2 supersymmetry. A stack of $D3$ -branes gives rise to an N=4 super Yang–Mills theory on their world–volume, provided the stack does not sit at an orbifold singularity. Hence, together with their world–volume gauge fields, the scalars C_i^3 are organized in $N = 4$ vector multiplets. The supersymmetry on the world–volume of a $D7$ -brane, which is wrapped around a generic supersymmetric 4-cycle of a CY–space, is N=2. The scalar $C_i^{7_i}$ describes one (complex) scalar of a vectormultiplet. However, on the torus T^6 , the world–volume theory on the $D7$ -branes preserves N=4 supersymmetry. This becomes clear because one obtains in addition the scalars $C_i^{7_j}$, $i \neq j$, which represent four real N=2 hypermultiplet scalars. Hence together with the N=2 vectormultiplet, the latter constitute one N=4 vectormultiplet.

However, N=2 and chiral N=1 fields come from the twisted sector. The twisted matter fields C^{37_a} originate from open strings stretched between the $D3$ - and $D7$ -branes from the a -th stack. Without 2-form fluxes on the internal coordinates of the $D7$ -brane these strings have DN -boundary conditions w.r.t. to those four coordinates. With non-vanishing 2-form fluxes on the $D7$ -brane world–volume, the DN -boundary conditions become mixed boundary conditions, with one open string end respecting the fluxes on the $D7$ -branes. Generically, these fields respect N=2 supersymmetry. However, there are twisted N=1 matter fields $C^{7_a 7_b}$ arising from open strings stretched between two different stacks a and b of $D7$ -branes.

Let us now move on to the low–energy effective action describing the dynamics of the various moduli fields encountered above. The encountered complex scalars S, T^j, U^j give rise to the closed string moduli space. Since these fields live in the bulk, they constitute

⁹ We assume the 4-cycle to be a direct product of two 2-tori. At any rate, since $h_{(1,1)} = h_{(2,2)} = 3$, these are the only 4-cycles in the orbifold under consideration.

the bulk–moduli space. The latter is a Kähler manifold, with the corresponding Kähler potential \widehat{K} given by [39]:

$$\kappa_4^2 \widehat{K} = -\ln(S - \bar{S}) - \sum_{i=1}^3 \ln(T^i - \bar{T}^i) - \sum_{i=1}^3 \ln(U^i - \bar{U}^i). \quad (3.1)$$

The untwisted open string moduli describe either the displacement transverse to the D –brane world–volume or the breaking of the gauge group by Wilson lines. It is justified to expand the (full) Kähler potential K and superpotential W around this minimum $C_i = 0$. In the case, that we have one stack of space–time filling $D3$ –branes and several stacks of $D7$ –branes wrapped around different 4–cycles (and being transverse to one torus $T^{2,j}$), the Kähler potential takes the following expansion (for large Kähler moduli, which corresponds to the supergravity approximation under consideration):

$$\begin{aligned} K(M, \bar{M}, C, \bar{C}) = & \widehat{K}(M, \bar{M}) + \sum_{i=1}^3 G_{C_i^3 \bar{C}_i^3}(M, \bar{M}) C_i^3 \bar{C}_i^3 \\ & + \sum_a \sum_{j=1}^3 \sum_{i=1}^3 G_{C_i^{7_a, j} \bar{C}_i^{7_a, j}}(M, \bar{M}) C_i^{7_a, j} \bar{C}_i^{7_a, j} \\ & + \sum_a G_{C^{37_a} \bar{C}^{37_a}}(M, \bar{M}) C^{37_a} \bar{C}^{37_a} \\ & + \sum_{a \neq b} G_{C^{7_a 7_b} \bar{C}^{7_a 7_b}}(M, \bar{M}) C^{7_a 7_b} \bar{C}^{7_a 7_b} + \mathcal{O}(C^4). \end{aligned} \quad (3.2)$$

Here, M collectively accounts for the closed string moduli fields S , T^i , U^i , and C for the open string moduli, *i.e.* matter fields. The index a denotes a particular stack of $D7$ –branes, which has a fixed transverse torus $T^{2,j}$. Furthermore, the holomorphic superpotential W takes the form:

$$\begin{aligned} W(M, C) = & \widehat{W}(M) + C_1^3 C_2^3 C_3^3 + \sum_a \sum_{j=1}^3 C_1^{7_a, j} C_2^{7_a, j} C_3^{7_a, j} \\ & + \sum_{I, J, K} Y_{IJK}(U^i) C_I C_J C_K + \mathcal{O}(C^4). \end{aligned} \quad (3.3)$$

Here, the C_I collectively account for all other combinations of matter fields, in particular the twisted matter fields $C^{37_a}, C^{7_a 7_b}$. The Yukawa couplings $Y_{IJK}(U^i)$, depending only on the complex structure moduli U^i , have been determined in *Ref.* [40]. In vacua without fluxes, we have $\widehat{W}(M) = 0$ as a result of the flatness of the closed string moduli. The closed string superpotential $\widehat{W}(M)$, which is induced by the 3–form flux has been already presented in *Eq.* (2.32). Obviously, it is only non–vanishing for $(0, 3)$ –flux.

3.1. Matter field metrics of untwisted open strings

The metrics $G_{C_i^3 \overline{C}_j^3}$ and $G_{C_i^{7,j} \overline{C}_i^{7,j}}$ for the untwisted matter fields C_i^3 and $C_i^{7,j}$ (of a particular $D7$ -brane stack a) may be obtained from the following differential equation [1]:

$$\partial_{\text{Im}\mathcal{T}^j} G_{C_i \overline{C}_i} = \frac{D^j + \overline{D}^j}{4\text{Im}\mathcal{T}^j} (1 - 2\delta^{ij}) G_{C_i \overline{C}_i} . \quad (3.4)$$

The latter is derived from a string scattering amplitude of two matter fields C_i, C_k and the closed string modulus \mathcal{T}^j . There is no coupling between matter fields referring to different planes, *i.e.* $G_{C_i \overline{C}_j} = 0$, $i \neq j$ as a result of internal charge conservation [1]. This fact justifies our ansatz for the expansion of Kähler potential (3.2) *ex post facto*. The “matrix” D depends on whether Dirichlet or Neumann boundary conditions are imposed on the open string fields attached to the D -brane. In other words, the matrix specifies, which of the matter fields $C_i^3, C_i^{7,j}$ we are considering in the scattering process. For untwisted $D7$ -brane matter fields $C_i^{7,j}$, the matrix D also encodes the wrapping and flux properties of the $D7$ -brane. The $D3$ -brane case is particularly simple. In that case, all six internal open string coordinates respect Dirichlet boundary conditions, *i.e.* $D^i = -1$, and the differential equation (3.4) yields:

$$G_{C_i^3 \overline{C}_i^3} \sim \alpha'^{-3/2} e^{-\phi_4} \frac{1}{\mathcal{U}^i - \overline{\mathcal{U}}^i} \sqrt{\frac{(\mathcal{T}^i - \overline{\mathcal{T}}^i)}{(\mathcal{T}^k - \overline{\mathcal{T}}^k)(\mathcal{T}^l - \overline{\mathcal{T}}^l)}} , \quad (i, k, l) = \overline{(1, 2, 3)} . \quad (3.5)$$

Expressed in the field-theory moduli (2.6) and (2.7), we obtain:

$$G_{C_i^3 \overline{C}_i^3} = \frac{-\kappa_4^{-2}}{(U^i - \overline{U}^i)(T^i - \overline{T}^i)} , \quad i = 1, 2, 3 . \quad (3.6)$$

The extra U -dependence comes from considering a four-point scattering amplitude [1]. Let us now move on to the untwisted $D7$ -matter fields $C_i^{7,j}$. For concreteness, let us consider the fields $C_i^{7,3}$, *i.e.* we shall discuss the case of a $D7$ -brane, which is transversal to the third torus $T^{2,3}$. In this specific case, we find:

$$\begin{aligned} D^1 &= \frac{\text{Im}\mathcal{T}^1 - if^1}{\text{Im}\mathcal{T}^1 + if^1} , \\ D^2 &= \frac{\text{Im}\mathcal{T}^2 - if^2}{\text{Im}\mathcal{T}^2 + if^2} , \\ D^3 &= -1 . \end{aligned} \quad (3.7)$$

After substituting this and integrating the differential equation, we obtain the following metrics:

$$\begin{aligned}
G_{C_1^{\tau,3}\overline{C}_1^{\tau,3}} &\sim e^{-\phi_4} \frac{1}{\mathcal{U}^1 - \overline{\mathcal{U}}^1} \sqrt{\frac{\mathcal{T}^1 - \overline{\mathcal{T}}^1}{(\mathcal{T}^2 - \overline{\mathcal{T}}^2)(\mathcal{T}^3 - \overline{\mathcal{T}}^3)}} \frac{|\text{Im}\mathcal{T}^2 + if^2|}{|\text{Im}\mathcal{T}^1 + if^1|} , \\
G_{C_2^{\tau,3}\overline{C}_2^{\tau,3}} &\sim e^{-\phi_4} \frac{1}{\mathcal{U}^2 - \overline{\mathcal{U}}^2} \sqrt{\frac{\mathcal{T}^2 - \overline{\mathcal{T}}^2}{(\mathcal{T}^1 - \overline{\mathcal{T}}^1)(\mathcal{T}^3 - \overline{\mathcal{T}}^3)}} \frac{|\text{Im}\mathcal{T}^1 + if^1|}{|\text{Im}\mathcal{T}^2 + if^2|} , \\
G_{C_3^{\tau,3}\overline{C}_3^{\tau,3}} &\sim e^{-\phi_4} \frac{1}{\mathcal{U}^3 - \overline{\mathcal{U}}^3} \sqrt{\frac{\mathcal{T}^3 - \overline{\mathcal{T}}^3}{(\mathcal{T}^1 - \overline{\mathcal{T}}^1)(\mathcal{T}^2 - \overline{\mathcal{T}}^2)}} |\text{Im}\mathcal{T}^1 + if^1| |\text{Im}\mathcal{T}^2 + if^2| .
\end{aligned} \tag{3.8}$$

Expressed through the moduli in the field theory basis, the metric reads

$$\begin{aligned}
G_{C_1^{\tau,3}\overline{C}_1^{\tau,3}} &= \frac{-\kappa_4^{-2}}{(U^1 - \overline{U}^1)(T^2 - \overline{T}^2)} \frac{|1 + i\tilde{f}^2|}{|1 + i\tilde{f}^1|} , \\
G_{C_2^{\tau,3}\overline{C}_2^{\tau,3}} &= \frac{-\kappa_4^{-2}}{(U^2 - \overline{U}^2)(T^1 - \overline{T}^1)} \frac{|1 + i\tilde{f}^1|}{|1 + i\tilde{f}^2|} , \\
G_{C_3^{\tau,3}\overline{C}_3^{\tau,3}} &= \frac{-\kappa_4^{-2}}{(U^3 - \overline{U}^3)(S - \overline{S})} |1 - \tilde{f}^1\tilde{f}^2| , \\
G_{C_i\overline{C}_k} &= 0, \quad i \neq k ,
\end{aligned} \tag{3.9}$$

where $\tilde{f}^i = \frac{f^i}{\text{Im}\mathcal{T}^i}$ is the physical 2-form flux. The other cases $G_{C_i^{\tau,j}\overline{C}_i^{\tau,j}}$ with $j = 1, 2$ are obtained from the above results by permuting fields. After putting our results together, the Kähler potential (3.2) for the untwisted closed string sector becomes up to second order in the open string matter fields

$$\begin{aligned}
\kappa_4^2 K(M, \overline{M}, C, \overline{C}) &= -\ln(S - \overline{S}) - \sum_{i=1}^3 \ln(T^i - \overline{T}^i) - \sum_{i=1}^3 \ln(U^i - \overline{U}^i) \\
&+ \sum_{i=1}^3 \frac{|C_i^3|^2}{(T^i - \overline{T}^i)(U^i - \overline{U}^i)} + \sum_a \sum_{i=1}^3 d_{ikl} \frac{|C_i^{\tau,a,i}|^2}{(S - \overline{S})(U^i - \overline{U}^i)} |1 - \tilde{f}^k\tilde{f}^l| \\
&+ \sum_a \sum_{j=1}^3 \sum_{i=1}^3 d_{ijk} \frac{|C_i^{\tau,a,j}|^2}{(T^k - \overline{T}^k)(U^i - \overline{U}^i)} \left| \frac{1 + i\tilde{f}^k}{1 + i\tilde{f}^i} \right| .
\end{aligned} \tag{3.10}$$

Here, we have introduced the tensor d_{ijk} which is 1 for (i, j, k) a permutation of $(1, 2, 3)$ and 0 otherwise. There is one comment in order. As already anticipated, the $D3$ -brane moduli C_j^3 , $j = 1, 2, 3$ describe scalars of an N=4 vector multiplet and the $D7$ -brane

moduli $C_i^{7,i}$ are N=2 vectormultiplet scalars. The N=4 vector multiplet of the $D3$ -brane may be split into one N=2 hypermultiplet in the adjoint and one N=2 vectormultiplet. The latter describes the relative position of the $D3$ -brane to that of the $D7$ -brane in the space transversal to the $D7$ -brane. Hence, for a given internal index i the fields $C_i^{7,i}$ and C_i^3 represent N=2 vectormultiplet scalars and we expect their metrics to be deducible from a common N=2 prepotential \mathcal{F} . Indeed, for the case of vanishing 2-form fluxes, from the N=2 trilinear prepotential \mathcal{F} of special geometry [41]:

$$\mathcal{F}(S, T^i, U^i, C_i^3, C_i^{7a,i}) = S [T^i U^i + \frac{1}{2} (C_i^3)^2] + \frac{1}{2} \sum_a T^i (C_i^{7a,i})^2 . \quad (3.11)$$

we derive the untwisted sector Kähler potential

$$-\ln \left[(S - \bar{S})(T^i - \bar{T}^i)(U^i - \bar{U}^i) + \frac{1}{2} (S - \bar{S})(C_i^3 - \bar{C}_i^3)^2 + \frac{1}{2} \sum_a (T^i - \bar{T}^i)(C_i^{7a,i} - \bar{C}_i^{7a,i})^2 \right] \quad (3.12)$$

describing to the order of $C\bar{C}$ a part of the second line of (3.10) (in the case without 2-form fluxes). In addition, from Eq. (3.12) there follows an H -term

$$H_{ii} = - \frac{\kappa_4^{-2}}{(U^i - \bar{U}^i)(T^i - \bar{T}^i)} \quad (3.13)$$

for the $D3$ -brane moduli C_i^3 . For the $D7$ -brane scalars $C_i^{7,i}$ the following H_{ii} -term is generated:

$$H_{ii} = - \frac{\kappa_4^{-2}}{(U^i - \bar{U}^i)(S - \bar{S})} . \quad (3.14)$$

However, in the more general case of non-vanishing 2-form fluxes and several stacks of $D7$ -branes wrapping different 4-cycles, the Kähler potential has to be given in the form (3.10). There is no obvious disentangling of moduli field according to the N=2 supersymmetry on a given $D7$ -brane. Hence in that general case we do not encounter H -terms, *i.e.* $H_{ii} = 0$.

3.2. Matter field metrics for 1/4 BPS brane configurations

Let us now come to the Kähler metrics of the twisted sector matter fields C^{7a7b} . These fields arise from massless open strings stretched between two different stacks a, b of $D7$ -branes of $D7$ -branes. These bosonic fields, arising from the open string NS -sector, build chiral multiplets, with their fermionic partners arising from the twisted R -ground state. Generically, the open strings respect only N=1 supersymmetry due to the different 2-form flux distributions on the $D7$ -branes. In the dual type IIA picture, these open strings are stretched between two intersecting $D6$ -branes, with angles θ_{ab}^j , $j = 1, 2, 3$. The matter field metrics for scalar matter fields arising from those open strings has been

calculated in *Ref.* [1]. Essentially, we have to translate these results into the type *IIB* picture with non-vanishing 2-form fluxes. For concreteness, let us consider the case of an open string “stretched” between two *D9*-branes $D9_a$ and $D9_b$ with the 2-form fluxes f_a^j, f_b^j , $j = 1, 2, 3$ on their internal world-volume. In that case the matter field metric takes the following form:

$$G_{C^{9_a 9_b} \overline{C}^{9_a 9_b}} = \kappa_4^{-2} \prod_{j=1}^3 (U^j - \overline{U}^j)^{-\theta_{ab}^j} \sqrt{\frac{\Gamma(\theta_{ab}^j)}{\Gamma(1 - \theta_{ab}^j)}} \quad , \quad \theta_{ab}^j \neq 0, 1 \quad . \quad (3.15)$$

Here the angle θ_{ab}^j , reminiscent from the type *IIA* description, encodes the two flux components on the different stacks a and b of *D9*-branes:

$$\theta_{ab}^j = \frac{1}{\pi} \left[\arctan \left(\frac{f_b^j}{\text{Im}(\mathcal{T}^j)} \right) - \arctan \left(\frac{f_a^j}{\text{Im}(\mathcal{T}^j)} \right) \right] \quad . \quad (3.16)$$

Generically, for two stacks of *D7*-branes a and b , which wrap different 4-cycles and whose 2-form fluxes fulfill (2.17), there is always a non-vanishing relative “flux” θ_{ab}^j in each plane, *i.e.* $\theta_{ab}^j \neq 0, 1$. In other words, these stacks are N=1 supersymmetric. Hence, in that case *Eq.* (3.15) directly applies and we obtain

$$G_{C^{\tau_a \tau_b} \overline{C}^{\tau_a \tau_b}} = \kappa_4^{-2} \prod_{j=1}^3 (U^j - \overline{U}^j)^{-\theta_{ab}^j} \sqrt{\frac{\Gamma(\theta_{ab}^j)}{\Gamma(1 - \theta_{ab}^j)}} \quad , \quad (3.17)$$

with θ_{ab}^j given in (3.16).

However, if $\theta_{ab}^j = 0, 1$ for one complex plane j , the two stacks a and b preserve N=2 supersymmetry and (3.15) does not directly apply to these cases. This case will be discussed in the next subsection.

3.3. Matter field metrics for 1/2 BPS brane configurations

In this subsection, we shall derive the metrics for matter fields arising from open strings stretched between two branes a and b , which preserve N=2 space-time supersymmetry. This case has not been discussed in [1] and a direct application of the formula (3.15), valid for the N=1 case, is not obvious¹⁰. Generically, these cases arise, if both branes a and b have vanishing fluxes $f^j = 0$ in one and the same internal complex plane j . In the dual type *IIA* picture, this corresponds to the relative angle $\theta_{ab}^j = 0, 1$ in that plane. In particular,

¹⁰ Note, that a similar problem arises in the case of one-loop gauge threshold corrections [42]. There, the one-loop correction arising from open string N=1 sectors is not directly related to the contribution stemming from the N=2 sectors.

this is true for the metrics $G_{C^{37_a}\overline{C}^{37_a}}$. In addition, two $D7$ -branes $D7_a$ and $D7_b$ without internal 2-form fluxes, one transversal to the torus $T^{2,a}$ and the other transversal to the torus $T^{2,b}$, are 1/2 BPS.

In the following, we shall assume for concreteness, that both branes have their vanishing 2-form fluxes in the third complex plane, *i.e.* $f_a^3 = 0 = f_b^3$. The two open string coordinates referring to this plane obey either pure Dirichlet or Neumann boundary conditions at both string ends. W.r.t. the other two planes, the branes a and b may carry the 2-form fluxes f_a^j, f_b^j , $j = 1, 2$ on their internal world-volume. The latter are assumed to fulfill the supersymmetry condition (2.17):

$$\frac{f_x^2}{\text{Im}(\mathcal{T}^2)} = -\frac{f_x^1}{\text{Im}(\mathcal{T}^1)} \quad , \quad x = a, b . \quad (3.18)$$

In the dual type IIA picture, the branes a and b describe two intersecting $D6$ -branes. Their relative angles $\theta_{ab}^j = \theta_b^j - \theta_a^j$ (with $\tan(\pi\theta_x^j) = \frac{f_x^j}{\text{Im}(\mathcal{T}^j)}$, $x = a, b$) are given by (3.16), for $j = 1, 2$. Furthermore, $\theta_{ab}^3 = 0$. In order, for the two branes to preserve $N=2$ supersymmetry, these relative angles have to fulfill:

$$\theta_{ab}^1 + \theta_{ab}^2 = 0 \text{ mod } 1 . \quad (3.19)$$

Massless open strings stretched between brane a and b give rise to $N=2$ matter fields from the twisted sector. In the (-1) -ghost picture, their vertex operator is given by [31]:

$$V_{C_\theta}^{(-1)}(z, k) = \lambda e^{-\phi(z)} \prod_{j=1}^2 s_{\theta^j}(z) \sigma_{-\theta^j}(z) e^{ik_\nu X^\nu(z)} . \quad (3.20)$$

The bosonic twist fields $\sigma_{-\theta^j}$ have conformal dimension $\frac{1}{2}\theta^j(1-\theta^j)$ and the spin fields have dimension $\frac{1}{2}(1-\theta^j)^2$. With this information and after imposing (3.19), it is straightforward to check, that the operator (3.20) has conformal dimension one. To derive the moduli dependence of the matter field metric $G_{C^{ab}\overline{C}^{ab}}$, we proceed similarly as in [1], *i.e.* we calculate the three-point amplitudes $\langle V_{C_\theta}^{(-1)} V_{\overline{C}_\theta}^{(-1)} V_{\mathcal{U}^j}^{(0,0)} \rangle$ and the four-point amplitudes $\langle V_{C_\theta}^{(-1)} V_{\overline{C}_\theta}^{(-1)} V_{\mathcal{T}^i}^{(0,0)} V_{\overline{\mathcal{T}}^j}^{(0,0)} \rangle$ in the dual type IIA picture, *i.e.* $C_\theta \simeq C^{ab}$. For the case $j = 3$, the internal part of the modulus vertex $V_{\mathcal{U}^j}^{(0,0)}$ decouples from the twist fields. Essentially, the amplitude $\langle V_{C_\theta}^{(-1)} V_{\overline{C}_\theta}^{(-1)} V_{\mathcal{U}^j}^{(0,0)} \rangle$ leads to the same contractions as in the case of two matter fields and one dilaton field, given in [1]. To this end, we find the differential equation (already translated into the type IIB picture):

$$\partial_{\mathcal{T}_2^3} G_{C^{ab}\overline{C}^{ab}} = \frac{\pm i}{\mathcal{T}^3 - \overline{\mathcal{T}}^3} . \quad (3.21)$$

The two signs depend on whether Neumann or Dirichlet boundary conditions are imposed on the open string coordinates from the third plane, respectively. Hence (3.21) gives rise to the following \mathcal{T}^3 -dependence:

$$G_{C^{ab}\overline{C}^{ab}} \sim (\mathcal{T}^3 - \overline{\mathcal{T}}^3)^{\pm \frac{1}{2}} . \quad (3.22)$$

On the other hand, for $j = 1, 2$, the amplitude $\langle V_{C_\theta}^{(-1)} V_{\overline{C}_\theta}^{(-1)} V_{\mathcal{U}^j}^{(0,0)} \rangle$ yields the same differential equation as found in [1] for each plane separately. Hence the total dependence on the Kähler moduli becomes

$$G_{C^{ab}\overline{C}^{ab}} \sim (\mathcal{T}^3 - \overline{\mathcal{T}}^3)^{\pm \frac{1}{2}} \prod_{j=1}^2 \sqrt{\frac{\Gamma(\theta_{ab}^j)}{\Gamma(1 - \theta_{ab}^j)}} , \quad (3.23)$$

with the angles given in Eq. (3.16). To determine the complex structure dependence of the metric $G_{C^{ab}\overline{C}^{ab}}$, we calculate the amplitude $\langle V_{C_\theta}^{(-1)} V_{\overline{C}_\theta}^{(-1)} V_{\mathcal{T}^i}^{(0,0)} V_{\overline{\mathcal{T}}^j}^{(0,0)} \rangle$ in the dual type *IIA* picture. Clearly, the amplitude is vanishing for $i \neq j$ due to internal charge conservation. Furthermore, for $i = 1, 2$, *i.e.* the \mathcal{T}^i -moduli are from those two planes, for which $\theta_{ab}^i \neq 0$, our calculation boils down to the case discussed in [1] and yields

$$\langle V_{C_\theta}^{(-1)} V_{\overline{C}_\theta}^{(-1)} V_{\mathcal{T}^i}^{(0,0)} V_{\overline{\mathcal{T}}^i}^{(0,0)} \rangle \sim \frac{st}{u} + s \theta^i + \dots , \quad i = 1, 2 , \quad (3.24)$$

with the three kinematic invariants s, t and u . The dots stand for higher orders in the space-time momentum, which are not relevant to us. On the other hand, for $i = 3$, we find:

$$\langle V_{C_\theta}^{(-1)} V_{\overline{C}_\theta}^{(-1)} V_{\mathcal{T}^i}^{(0,0)} V_{\overline{\mathcal{T}}^i}^{(0,0)} \rangle \sim \frac{st}{u} + \dots , \quad i = 3 . \quad (3.25)$$

To the order in the momentum, which is displayed on the r.h.s. of both equations (3.24) and (3.25), both sides have to reproduce the σ -model result $G_{C^{ab}\overline{C}^{ab}} G_{\mathcal{T}^i \overline{\mathcal{T}}^i} \frac{st}{u} + s R_{C^{ab}\overline{C}^{ab} \mathcal{T}^i \overline{\mathcal{T}}^i}$. This information allows us to completely fix the \mathcal{T}^i -dependence of the matter metric $G_{C^{ab}\overline{C}^{ab}}$ in the *T*-dual type *IIA* picture.

Finally, after putting all results together, we obtain the metric of massless matter fields originating from a 1/2 BPS system of branes in type *IIB* :

$$G_{C^{ab}\overline{C}^{ab}} = \alpha'^{-(1 \pm \frac{1}{2})} e^{-\phi_4} (\mathcal{T}^3 - \overline{\mathcal{T}}^3)^{\pm \frac{1}{2}} \prod_{j=1}^2 (\mathcal{U}^j - \overline{\mathcal{U}}^j)^{-\theta^j} \sqrt{\frac{\Gamma(\theta^j)}{\Gamma(1 - \theta^j)}} . \quad (3.26)$$

From this expression, we can deduce various special cases¹¹, which are not captured by the formula (3.15). The latter is valid for 1/4 BPS systems. For a space–time filling $D3$ –brane and a $D7$ –brane, transversal to the third torus, we get¹²:

$$\begin{aligned} G_{C^{373}\bar{C}^{373}} &= 2^{1/2} i^{-1/2} \alpha'^{-1/2} e^{-\phi_4} (\mathcal{T}^3 - \bar{\mathcal{T}}^3)^{-1/2} \frac{1}{(\mathcal{U}^1 - \bar{\mathcal{U}}^1)^{1/2} (\mathcal{U}^2 - \bar{\mathcal{U}}^2)^{1/2}} \\ &= \frac{\kappa_4^{-2}}{(T^1 - \bar{T}^1)^{1/2} (T^2 - \bar{T}^2)^{1/2}} \frac{1}{(U^1 - \bar{U}^1)^{1/2} (U^2 - \bar{U}^2)^{1/2}} . \end{aligned} \quad (3.29)$$

In the case, that on the internal $D7$ –brane world volume non–trivial 2–form fluxes f^1, f^2

¹¹ Although not relevant to us in this article, for completeness, let us also discuss the special cases arising in type IIB orientifolds with $D9$ – and $D5$ –branes. First, we consider a $D5$ –brane, wrapped around the third torus $T^{2,j}$ and a $D9$ –brane wrapped around the full six–torus T^6 . This system, which is 1/2 BPS, preserves $N=2$ space–time supersymmetry. W.r.t. the third torus, open strings have Neumann boundary conditions. On the other hand, in the dual type IIA picture, the two branes intersect at the angles $\pi/2$ within the other two internal planes, *i.e.* $\theta_{ab}^j = \frac{1}{2}$, $j = 1, 2$. After recalling the relation between the moduli fields \mathcal{T}^j in the string basis vs. field–theory basis, namely $\text{Im}(S) = (2\pi)^{-1} \alpha'^{-3/2} e^{-\phi_4} \sqrt{T_2^1 T_2^2 T_2^3}$ and $\text{Im}(T^j) = (2\pi)^{-1} \alpha'^{1/2} e^{-\phi_4} \sqrt{\frac{T_2^j}{T_2^k T_2^l}}$, $(j, k, l) = \overline{(1, 2, 3)}$ for type IIB orientifolds with $D9$ and $D5$ –branes, we obtain (with $\kappa_4^{-2} = \frac{e^{-2\phi_4}}{\pi \alpha'}$):

$$\begin{aligned} G_{C^{953}\bar{C}^{953}} &= 2^{-1/2} i^{-3/2} \alpha'^{-3/2} e^{-\phi_4} (\mathcal{T}^3 - \bar{\mathcal{T}}^3)^{1/2} \frac{1}{(\mathcal{U}^1 - \bar{\mathcal{U}}^1)^{1/2} (\mathcal{U}^2 - \bar{\mathcal{U}}^2)^{1/2}} \\ &= \frac{\kappa_4^{-2}}{(T^1 - \bar{T}^1)^{1/2} (T^2 - \bar{T}^2)^{1/2}} \frac{1}{(U^1 - \bar{U}^1)^{1/2} (U^2 - \bar{U}^2)^{1/2}} . \end{aligned} \quad (3.27)$$

Furthermore, for two $D5$ –branes, with one wrapping the torus $T^{2,1}$ and the other one wrapping the torus $T^{2,2}$ we have pure Dirichlet boundary conditions w.r.t the open string coordinates from the third plane. Again, in the dual type IIA picture, the two branes intersect at the angles $\pi/2$ within the other two internal planes, *i.e.* $\theta_{ab}^j = \frac{1}{2}$, $j = 1, 2$. Hence, from *Eq.* (3.26) we deduce:

$$\begin{aligned} G_{C^{5152}\bar{C}^{5152}} &= 2^{1/2} i^{-1/2} \alpha'^{-1/2} e^{-\phi_4} (\mathcal{T}^3 - \bar{\mathcal{T}}^3)^{-1/2} \frac{1}{(\mathcal{U}^1 - \bar{\mathcal{U}}^1)^{1/2} (\mathcal{U}^2 - \bar{\mathcal{U}}^2)^{1/2}} \\ &= \frac{\kappa_4^{-2}}{(S - \bar{S})^{1/2} (T^3 - \bar{T}^3)^{1/2}} \frac{1}{(U^1 - \bar{U}^1)^{1/2} (U^2 - \bar{U}^2)^{1/2}} . \end{aligned} \quad (3.28)$$

¹² We use the translation rules (2.6) to go from the string–basis to the field–theory basis.

satisfying (3.18) are turned on, we obtain from Eq. (3.26):

$$\begin{aligned} G_{C^{37_3}\bar{C}^{37_3}} &= \frac{\kappa_4^{-2}}{(T^1 - \bar{T}^1)^{1/2}(T^2 - \bar{T}^2)^{1/2}} \prod_{j=1}^2 (U^j - \bar{U}^j)^{-\theta^j} \sqrt{\frac{\Gamma(\theta^j)}{\Gamma(1 - \theta^j)}} \\ &= \frac{\kappa_4^{-2}}{(T^1 - \bar{T}^1)^{1/2}(T^2 - \bar{T}^2)^{1/2}} \frac{1}{(U^1 - \bar{U}^1)^{\theta^1} (U^2 - \bar{U}^2)^{\theta^2}} \end{aligned} \quad (3.30)$$

The last equation follows from the N=2 supersymmetry condition (3.19), *i.e.* $\theta_{ab}^1 = 1 - \theta_{ab}^2 \bmod 1$. Finally, the special case of two $D7$ -branes $D7_1$ and $D7_2$ without internal 2-form fluxes, one transversal to the torus $T^{2,1}$ and the other transversal to the torus $T^{2,2}$, are 1/2 BPS. This corresponds to the case $\theta_{ab}^j = 1/2$, $j = 1, 2$ and $\theta_{ab}^3 = 0$, already discussed in Eq. (3.28) for a system of two $D5$ -branes. The metric of the corresponding open string matter field becomes:

$$\begin{aligned} G_{C^{7_1 7_2}\bar{C}^{7_1 7_2}} &= 2^{-1/2} i^{-3/2} \alpha'^{-1/2} e^{-\phi_4} (\mathcal{T}^3 - \bar{\mathcal{T}}^3)^{1/2} \frac{1}{(\mathcal{U}^1 - \bar{\mathcal{U}}^1)^{1/2} (\mathcal{U}^2 - \bar{\mathcal{U}}^2)^{1/2}} \\ &= \frac{\kappa_4^{-2}}{(S - \bar{S})^{1/2}(T^3 - \bar{T}^3)^{1/2}} \frac{1}{(U^1 - \bar{U}^1)^{1/2} (U^2 - \bar{U}^2)^{1/2}} . \end{aligned} \quad (3.31)$$

To conclude, in this subsection we have derived the metric for massless matter fields originating from open strings stretched between a space-time filling $D3$ -brane and a $D7$ -brane, transversal to the first torus $T^{2,1}$ with the 2-form fluxes f^2, f^3 on its internal world-volume:

$$G_{C^{37_1}\bar{C}^{37_1}} = \frac{\kappa_4^{-2}}{(T^2 - \bar{T}^2)^{\frac{1}{2}} (T^3 - \bar{T}^3)^{\frac{1}{2}}} \frac{1}{(U^2 - \bar{U}^2)^{\theta^2} (U^3 - \bar{U}^3)^{\theta^3}} \quad (3.32)$$

for the matter field metric referring to an open string stretched between a $D3$ -brane and a $D7$ -brane. The latter is located transversal to the first complex plane and carries the 2-form fluxes f^2, f^3 on its (internal) world-volume. The fluxes, which fulfill (2.17), are encoded in the angles θ^j :

$$\theta^j = \frac{1}{\pi} \arctan \left(\frac{f^j}{\text{Im}(\mathcal{T}^j)} \right) \quad , \quad j = 2, 3 . \quad (3.33)$$

3.4. Gauge kinetic function

Finally, the holomorphic gauge kinetic function, which encodes the gauge coupling of a $D7$ -brane, wrapped around the 4-cycle $T^{2,k} \times T^{2,l}$ [with wrapping numbers m^k, m^l and the supersymmetric 2-form fluxes f^k, f^l (*cf.* condition (2.17))] is:

$$f_{D7_j}(S, T^j) = |m^k m^l| (T^j - \alpha'^{-2} f^k f^l S) \quad , \quad (j, k, l) = \overline{(1, 2, 3)} . \quad (3.34)$$

Furthermore, the gauge sector of a space-time filling $D3$ -brane is encoded in the holomorphic function:

$$f_{D3}(S) = S . \quad (3.35)$$

Note, that the gauge couplings of the $D3$ -brane and the various $D7$ -branes are derived from the common holomorphic N=2 prepotential

$$\begin{aligned} \mathcal{F}(S, T^i, U^i, C_i^3, C_i^{7a,i}) = & S \left[\sum_{i=1}^3 T^i U^i + \frac{1}{2} (C_i^3)^2 \right] \\ & + \frac{1}{2} \sum_a \sum_{\substack{(j,k,l) \\ =(1,2,3)}} (T^j - \alpha'^{-2} f^k f^l S) (C_j^{7a,j})^2 \end{aligned} \quad (3.36)$$

as second derivative w.r.t. to the N=2 untwisted moduli fields $C_j^3, C_j^{7a,j}$ describing vector multiplet scalars, *i.e.* $g_{D7a,j}^{-2} = \text{Im} \frac{\partial^2 \mathcal{F}}{\partial (C_j^{7a,j})^2}$ and $g_{D3}^{-2} = \text{Im} \frac{\partial^2 \mathcal{F}}{\partial (C_j^3)^2}$. This represents a generalization of the expression (3.11) including all three complex planes $i = 1, 2, 3$. This indicates, that the gauge couplings still obey N=2 supersymmetry, even though the whole $D3/D7$ -brane configuration is only N=1 supersymmetric. In particular, even in the case of vanishing 2-form fluxes, the kinetic energy terms of the various bulk and brane moduli fields presented before, cannot be derived from (3.36).

4. The effective action with 3-form fluxes turned on

4.1. Superpotential, F -terms and scalar potential

We now want to obtain the low energy action with fluxes turned on. First, we will look at those quantities that can be derived in the bulk. The F -terms can be calculated from the Kähler potential (3.1) and the superpotential (2.32), they only feel the bulk:

$$\overline{F}^{\overline{I}} = e^{\kappa_4^2 \widehat{K}/2} \widehat{K}^{\overline{I}J} (\partial_J \widehat{W} + \kappa_4^2 \widehat{W} \partial_J \widehat{K}) , \quad (4.1)$$

where the I, J are taken to run over the dilaton S , the complex structure moduli U^i and the Kähler moduli T^i . With this, we can now calculate the scalar potential:

$$\widehat{V} = \widehat{K}_{I\overline{J}} F^I \overline{F}^{\overline{J}} - 3 e^{\kappa_4^2 \widehat{K}} \kappa_4^2 |\widehat{W}|^2 . \quad (4.2)$$

With what we learned in section 2, it is now easy to write down the superpotential explicitly for our case of $T^2 \times T^2 \times T^2$:

$$\begin{aligned} \frac{1}{\lambda} \widehat{W} = & (a^0 - S c^0) U^1 U^2 U^3 - \{ (a^1 - S c^1) U^2 U^3 + (a^2 - S c^2) U^1 U^3 + (a^3 - S c^3) U^1 U^2 \} \\ & - \sum_{i=1}^3 (b_i - S d_i) U^i - (b_0 - S d_0) . \end{aligned} \quad (4.3)$$

The explicit expressions for the F -terms are the following:

$$\begin{aligned}
\overline{F}^{\overline{S}} &= (S - \overline{S})^{1/2} \prod_{i=1}^3 (T^i - \overline{T}^i)^{-1/2} \prod_{i=1}^3 (U^i - \overline{U}^i)^{-1/2} \kappa_4^2 \frac{\lambda}{(2\pi)^2 \alpha'} \int \overline{G}_3 \wedge \Omega \\
&= \lambda \kappa_4^2 (S - \overline{S})^{1/2} \prod_{i=1}^3 (T^i - \overline{T}^i)^{-1/2} \prod_{i=1}^3 (U^i - \overline{U}^i)^{-1/2} \times \{ (a^0 - \overline{S}c^0) U^1 U^2 U^3 - \\
&\quad - [(a^1 - \overline{S}c^1) U^2 U^3 + (a^2 - \overline{S}c^2) U^1 U^3 + (a^3 - \overline{S}c^3) U^1 U^2] \\
&\quad - \sum_{i=1}^3 (b_i - \overline{S}d_i) U^i - (b_0 - \overline{S}d_0) \} , \\
\overline{F}^{\overline{T}^i} &= (S - \overline{S})^{-1/2} (T^i - \overline{T}^i)^{1/2} (T^j - \overline{T}^j)^{-1/2} (T^k - \overline{T}^k)^{-1/2} \prod_{i=1}^3 (U^i - \overline{U}^i)^{-1/2} \kappa_4^2 \widehat{W} \\
&= \lambda \kappa_4^2 (S - \overline{S})^{-1/2} (T^i - \overline{T}^i)^{1/2} (T^j - \overline{T}^j)^{-1/2} (T^k - \overline{T}^k)^{-1/2} \prod_{i=1}^3 (U^i - \overline{U}^i)^{-1/2} \times \\
&\quad \times \{ (a^0 - \overline{S}c^0) U^1 U^2 U^3 - [(a^1 - \overline{S}c^1) U^2 U^3 + (a^2 - \overline{S}c^2) U^1 U^3 + (a^3 - \overline{S}c^3) U^1 U^2] \\
&\quad - \sum_{i=1}^3 (b_i - \overline{S}d_i) U^i - (b_0 - \overline{S}d_0) \} , \\
\overline{F}^{\overline{U}^i} &= (S - \overline{S})^{-1/2} (U^i - \overline{U}^i)^{1/2} (U^j - \overline{U}^j)^{-1/2} (U^k - \overline{U}^k)^{-1/2} \prod_{l=1}^3 (T^l - \overline{T}^l)^{-1/2} \times \\
&\quad \times \kappa_4^2 \frac{\lambda}{(2\pi)^2 \alpha'} \int G_3 \wedge \omega_{A_i} , \quad i \neq j \neq k , \text{ e.g. :} \\
\overline{F}^{\overline{U}^1} &= \lambda \kappa_4^2 (S - \overline{S})^{-1/2} (U^1 - \overline{U}^1)^{1/2} (U^j - \overline{U}^j)^{-1/2} (U^k - \overline{U}^k)^{-1/2} \prod_{l=1}^3 (T^l - \overline{T}^l)^{-1/2} \times \\
&\quad \times \{ ((a^0 - \overline{S}c^0) \overline{U}^1 U^2 U^3 - [(a^1 - \overline{S}c^1) U^2 U^3 + (a^2 - \overline{S}c^2) \overline{U}^1 U^3 + (a^3 - \overline{S}c^3) \overline{U}^1 U^2] \\
&\quad - [(b_1 - \overline{S}d_1) \overline{U}^1 + (b_2 - \overline{S}d_2) U^2 + (b_3 - \overline{S}d_3) U^3] - (b_0 - \overline{S}d_0) \} .
\end{aligned} \tag{4.4}$$

By looking at the F -terms, we see immediately that we only have a non-zero $F^{\overline{S}}$ if G_3 has a $(3,0)$ -component. For $F^{\overline{T}^i}$ to be non-zero, G_3 has to have a non-zero $(0,3)$ -component. For the $F^{\overline{U}^i}$ to be non-zero, G_3 must have a $(1,2)$ -component.

Now we are able to compute the expression for the scalar potential (4.2). The part

coming from the T -moduli cancels with $-3e^{\widehat{K}\kappa_4^2}\kappa_4^2|\widehat{W}|^2$, so we are left with

$$\begin{aligned}\widehat{V} &= \partial_S \partial_{\overline{S}} \widehat{K} F^S \overline{F}^{\overline{S}} + \sum_{i=1}^3 \partial_{U_i} \partial_{\overline{U}_i} \widehat{K} F^{U_i} \overline{F}^{\overline{U}_i} \\ &= \frac{\lambda^2 \kappa_4^2}{(2\pi)^4 \alpha'^2} \left(|S - \overline{S}| \prod_{i=1}^3 |T^i - \overline{T}^i| \prod_{i=1}^3 |U^i - \overline{U}^i| \right)^{-1} \left(\left| \int \overline{G}_3 \wedge \Omega \right|^2 + \sum_{i=1}^3 \left| \int G_3 \wedge \omega_{A_i} \right|^2 \right).\end{aligned}\tag{4.5}$$

We can see immediately that \widehat{V} is zero unless G_3 has a $(3, 0)$ - or a $(1, 2)$ -part, i.e. is IASD. We also see immediately, that for IASD-fluxes, \widehat{V} is strictly positive. When we express this through the complex coefficients, this formula looks even nicer:

$$\widehat{V} = \lambda^2 \kappa_4^2 \frac{\prod_{i=1}^3 |U^i - \overline{U}^i|}{|S - \overline{S}| \prod_{i=1}^3 |T^i - \overline{T}^i|} \sum_{j=0}^3 |B^j|^2.\tag{4.6}$$

This ties in neatly with our observation from section 2: Let us examine eq. (2.29) : Expressed with our complex coefficients, we find

$$\int G_3 \wedge \star_6 \overline{G}_3 \propto 2 \sum_{i=0}^3 |B^i|^2 + \left(\sum_{i=0}^3 |A^i|^2 - \sum_{i=0}^3 |B^i|^2 \right),\tag{4.7}$$

where the second term is obviously proportional to N_{flux} , whereas the first part corresponds to V_{flux} , which is the contribution to the scalar potential coming from the F -terms, which is exactly, what we have calculated above.

The gravitino mass is given by

$$\begin{aligned}m_{3/2} &= e^{\kappa_4^2 \widehat{K}/2} \kappa_4^2 \widehat{W} = \lambda \kappa_4^2 (S - \overline{S})^{-1/2} \prod_{i=1}^3 (T^i - \overline{T}^i)^{-1/2} \prod_{i=1}^3 (U^i - \overline{U}^i)^{-1/2} \times \\ &\quad \times \{ (a^0 - S c^0) U^1 U^2 U^3 - [(a^1 - S c^1) U^2 U^3 + (a^2 - S c^2) U^1 U^3 + (a^3 - S c^3) U^1 U^2] \\ &\quad - \sum_{i=1}^3 (b_i - S d_i) U^i - (b_0 - S d_0) \}.\end{aligned}\tag{4.8}$$

4.2. Soft SUSY breaking terms

The effective low energy supergravity potential in the standard limit with $M_{\text{Pl}} \rightarrow \infty$ with $m_{3/2}$ fixed is for $N = 1$ supersymmetry [18,19]:

$$V^{\text{eff}} = \frac{1}{2} D^2 + G^{C_I \overline{C}_I} |\partial_I W^{(\text{eff})}|^2 + m_{H, \text{soft}}^2 C_I \overline{C}_I + \frac{1}{3} A_{IJK} C_I C_J C_K + \text{h.c.},\tag{4.9}$$

with

$$\begin{aligned} D &= -g_I \kappa_4^2 G_{C_I \bar{C}_I} C_I \bar{C}_I, \\ W^{(\text{eff})} &= \frac{1}{3} e^{\kappa_4^2 \hat{K}/2} Y_{IJK} C_I C_J C_K. \end{aligned} \quad (4.10)$$

The C_I are taken to run over the C_i^3 , $C_i^{7a,j}$, $C^{37,a}$, C^{7a7b} , where $i = 1, 2, 3$, a and b run over the stacks of branes, and j runs over the torus not being wrapped. Furthermore, g_I is the gauge coupling, which is related to the gauge kinetic function $f_I(M)$ by $g_I^{-2} = \text{Im}(f_I(M))$. The respective gauge kinetic functions are given in (3.34), (3.35).

The diagonal structure of our metrics already results in some simplifications, for example the purely diagonal structure of the scalar mass matrix. The fact that we have $H_{ij} = 0$ results in even more drastic simplifications: In our case, no B -term $B_{IJ} C_I C_J$ appears in the effective scalar potential, and also no μ -term $\frac{1}{2} \mu_{IJ} C_I C_J$ is generated in $W^{(\text{eff})}$.

The soft supersymmetry breaking terms are

$$\begin{aligned} m_{I\bar{I}, \text{soft}}^2 &= \kappa_4^2 [(|m_{3/2}|^2 + \kappa_4^2 \hat{V}) G_{C_I \bar{C}_I} - F^\rho \bar{F}^{\bar{\sigma}} R_{\rho \bar{\sigma} I \bar{I}}], \\ A_{IJK} &= F^\rho D_\rho (e^{\kappa_4^2 \hat{K}/2} Y_{IJK}), \end{aligned} \quad (4.11)$$

where the Greek indices are running over S , T^i , U^i and

$$\begin{aligned} R_{\rho \bar{\sigma} I \bar{I}} &= \frac{\partial^4 K}{\partial C_I \partial \bar{C}_I \partial M_\rho \partial \bar{M}_\sigma} - \frac{\partial^3 K}{\partial C_I \partial M_\rho \partial \bar{C}_K} G^{\bar{C}^\kappa C^\kappa} \frac{\partial^3 K}{\partial \bar{C}_I \partial \bar{M}_\sigma \partial C_K}, \\ D_\rho (e^{\kappa_4^2 \hat{K}/2} Y_{IJK}) &= \partial_\rho (e^{\kappa_4^2 \hat{K}/2} Y_{IJK}) + \frac{1}{2} \kappa_4^2 \hat{K}_\rho (e^{\kappa_4^2 \hat{K}/2} Y_{IJK}) \\ &\quad - e^{\kappa_4^2 \hat{K}/2} G^{\bar{C}_I C_I} \partial_\rho G_{C_I (\bar{C}_I} Y_{JK) I}. \end{aligned} \quad (4.12)$$

The gaugino mass is

$$m_{gI} = F^\rho \partial_\rho \log(\text{Im} f_I), \quad (4.13)$$

$f_I(M)$ being the gauge kinetic function.

Explicit calculation of the terms:

We first look at $W^{(\text{eff})}$. We find

$$W^{(\text{eff})} = \frac{1}{3} [(S - \bar{S}) \prod_{i=1}^3 (T^i - \bar{T}^i) \prod_{i=1}^3 (U^i - \bar{U}^i)]^{-1/2} Y_{IJK} C_I C_J C_K. \quad (4.14)$$

From eq. (3.3), we know that $Y_{IJK} = \epsilon_{IJK}$ in the case of the untwisted matter fields $C_i^3, C_i^{7,a}$ and the combination $\sum_a C^{7a,1} C^{7a,2} C^{7a,3} C^{7a,1}$.

Before we can calculate the expressions for the scalar masses $m_{I\bar{I},\text{soft}}$, we must first find the explicit expressions for the curvature tensor. This is done in Appendix B. We get

$$\begin{aligned}
(m_{i\bar{i}}^{33})^2 &= \kappa_4^2 \left[(|m_{3/2}|^2 + \kappa_4^2 \widehat{V}) G_{C_i^3 \bar{C}_i^3} - |F^{U^i}|^2 R_{U^i \bar{U}^i i\bar{i}}^3 - |F^{T^i}|^2 R_{T^i \bar{T}^i i\bar{i}}^3 \right], \\
(m_{i\bar{i}}^{7,j})^2 &= \kappa_4^2 \left[(|m_{3/2}|^2 + \kappa_4^2 \widehat{V}) G_{C_i^{7,j} \bar{C}_i^{7,j}} - \sum_{M,N} F^M \bar{F}^{\bar{N}} R_{M\bar{N} i\bar{i}}^{7,j} \right], \\
(m^{37_a})^2 &= \kappa_4^2 \left[(|m_{3/2}|^2 + \kappa_4^2 \widehat{V}) G_{C^{37_a} \bar{C}^{37_a}} - \sum_{M,N} F^M \bar{F}^{\bar{N}} R_{M\bar{N}}^{37_a} \right], \\
(m^{7a7b})^2 &= \kappa_4^2 \left[(|m_{3/2}|^2 + \kappa_4^2 \widehat{V}) G_{C^{7a7b} \bar{C}^{7a7b}} - \sum_{M,N} F^M \bar{F}^{\bar{N}} R_{M\bar{N}}^{7a7b} \right],
\end{aligned} \tag{4.15}$$

where M, N run over S, T^i, U^i . The trilinear coupling is

$$\begin{aligned}
A_{IJK} &= i \prod_M (M - \bar{M})^{-1} \frac{\lambda \kappa_4^2}{(2\pi)^2 \alpha'} \left\{ Y_{IJK} \int G_3 \wedge \bar{\Omega} + 3 Y_{IJK} \int \bar{G}_3 \wedge \bar{\Omega} \right. \\
&\quad \left. + \sum_i \int \bar{G}_3 \wedge \bar{\omega}_{A_i} [Y_{IJK} - (U^i - \bar{U}^i) \partial_{U^i} Y_{IJK}] \right\} \\
&\quad - i \prod_M (M - \bar{M})^{-1/2} F^\rho G^{\bar{C}_I C_I} \partial_\rho G_{C_I}(\bar{C}_I Y_{JK})_I.
\end{aligned} \tag{4.16}$$

The term $-\sum_i \int \bar{G}_3 \wedge \bar{\omega}_{A_i} (U^i - \bar{U}^i) \partial_{U^i} Y_{IJK}$ appears, because general Y_{IJK} may depend on the complex structure moduli.

Note the case where the I, J, K refer to the 3-brane matter fields C_i^3 : Then the last term cancels the terms $3 \int \bar{G}_3 \wedge \bar{\Omega}$ and $\sum_i \int \bar{G}_3 \wedge \bar{\omega}_{A_i}$, and we are left with

$$A_{IJK} = i \epsilon_{IJK} \prod_M (M - \bar{M})^{-1} \frac{\lambda \kappa_4^2}{(2\pi)^2 \alpha'} \int G_3 \wedge \bar{\Omega},$$

i.e. we only get a trilinear coupling from the $(3,0)$ -flux, which agrees with the results of [20,21]. This is not true for the other matter fields, as their metrics have a more complicated dependence on the moduli.

The gauge couplings have been given in Eq. (3.34) and (3.35). Through them, we obtain the gaugino masses:

$$\begin{aligned}
m_{g,D7j} &= F^S \frac{-\alpha'^{-2} f^k f^l}{(T^j - \bar{T}^j) - \alpha'^{-2} f^k f^l (S - \bar{S})} + F^{T^j} \frac{1}{(T^j - \bar{T}^j) - \alpha'^{-2} f^k f^l (S - \bar{S})}, \\
m_{g,D3} &= F^S \frac{1}{(S - \bar{S})} = -i \prod_M (M - \bar{M})^{-1/2} \frac{\lambda \kappa_4^2}{(2\pi)^2 \alpha'} \int G_3 \wedge \bar{\Omega},
\end{aligned} \tag{4.17}$$

with $k \neq l \neq j$, j being the torus not wrapped by the 7-brane.

We shall now examine the soft terms for the different flux components turned on separately. When examining the effective potential, we find that the D -term and $W^{(\text{eff})}$ do not depend on the 3-form fluxes.

(i) $(2,1)$ -flux

The $(2,1)$ -flux fulfills the ISD condition $\star_6 G_3 = i G_3$ w.r.t. the Hodge operation in the internal (compact) six-dimensional compactification manifold X_6 . For this flux component, the superpotential, the scalar potential in the bulk and all the F -terms are zero, as well as the gravitino mass $m_{3/2}$, the soft terms and the gaugino mass m_g . This is no surprise as we know the $(2,1)$ -component to preserve supersymmetry.

(ii) $(0,3)$ -flux

The $(0,3)$ -flux is imaginary self-dual as well. It is the only flux component with $\widehat{W} \neq 0$. This leads to non-zero F^{T^i} and non-zero gravitino mass $m_{3/2} = e^{\kappa_4^2 \widehat{K}/2} \kappa_4^2 \widehat{W}$. The scalar potential in the bulk vanishes, therefore

$$\begin{aligned}
(m_{\tilde{i}\tilde{i}}^{33})^2 &= \kappa_4^2 \left(|m_{3/2}|^2 G_{C_i^3 \bar{C}_i^3} - |F^{T^i}|^2 R_{T^i \bar{T}^i \tilde{i}\tilde{i}}^3 \right), \\
(m_{\tilde{i}\tilde{i}}^{7,j})^2 &= \kappa_4^2 \left(|m_{3/2}|^2 G_{C_i^7 \bar{C}_i^{7,j}} - \sum_{k,l} F^{T^k} \bar{F}^{\bar{T}^l} R_{T^k \bar{T}^l \tilde{i}\tilde{i}}^{7,j} \right), \\
(m^{37_a})^2 &= \kappa_4^2 \left(|m_{3/2}|^2 G_{C^{37_a} \bar{C}^{37_a}} - \sum_{k,l} F^{T^k} \bar{F}^{\bar{T}^l} R_{T^k \bar{T}^l}^{37_a} \right), \\
(m^{7a7b})^2 &= \kappa_4^2 \left(|m_{3/2}|^2 G_{C^{7a7b} \bar{C}^{7a7b}} - \sum_{k,l} F^{T^k} \bar{F}^{\bar{T}^l} R_{T^k \bar{T}^l}^{7a7b} \right),
\end{aligned} \tag{4.18}$$

The scalar mass term for the 3-brane deserves our special attention. After inspection of our findings of appendix B, we find that $|F^{T^i}|^2 R_{T^i \bar{T}^i \tilde{i}\tilde{i}}^3 = |m_{3/2}|^2 G_{C_i^3 \bar{C}_i^3}$, therefore

$$(m_{\tilde{i}\tilde{i}}^{33})^2 = 0.$$

The same would be true for the scalars on the 7-branes without 2-form fluxes. Turning on 2-form flux destroys the simple structure of the curvature tensors and thus generates scalar mass terms.

The trilinear coupling becomes

$$\begin{aligned}
A_{IJK} &= 3i Y_{IJK} \prod_M (M - \bar{M})^{-1} \frac{\lambda \kappa_4^2}{(2\pi)^2 \alpha'} \int \bar{G}_3 \wedge \bar{\Omega} \\
&\quad - i \prod_M (M - \bar{M})^{-1/2} \sum_{i=1}^3 F^{T^i} G^{\bar{C}_I C_I} \partial_{T^i} G_{C_I} (\bar{C}_I Y_{JK})_I.
\end{aligned} \tag{4.19}$$

For I, J, K referring to C_i^3 , the trilinear coupling is zero. The gaugino masses are

$$m_{g,D7,j} = \frac{F^{T^j}}{(T^j - \bar{T}^j) - \alpha'^2 f^k f^l (S - \bar{S})}.$$

To have an explicit example, we will substitute our solution for the $(0,3)$ -flux (2.46) for $U^1 = U^2 = U^3 = S = i$ into the above formulas. The superpotential is $\widehat{W} = 8\lambda(d_3 - id_0)$. The F -terms become

$$F^{T^i} = -2\lambda \kappa_4^2 (d_3 + id_0)(T^i - \bar{T}^i)^{1/2}(T^j - \bar{T}^j)^{-1/2}(T^k - \bar{T}^k)^{-1/2}, \quad i \neq j \neq k.$$

The gravitino mass is

$$|m_{3/2}|^2 = 4\lambda^2 \kappa_4^4 \prod_i |T^i - \bar{T}^i|^{-1}(d_3^2 + d_0^2).$$

The gaugino masses are

$$m_{g,D7,j} = -2\lambda \kappa_4^2 (d_3 + id_0) \frac{(T^j - \bar{T}^j)^{1/2}(T^k - \bar{T}^k)^{-1/2}(T^l - \bar{T}^l)^{-1/2}}{(T^j - \bar{T}^j) - 2i\alpha'^2 f^k f^l}.$$

(iii) $(1,2)$ -flux

Here, only $F^{U^i} \neq 0$. The gravitino mass $m_{3/2}$ vanishes again and $m_g = 0$. For the remaining terms, we find

$$\begin{aligned} \widehat{V} &= -\kappa_4^{-2} \sum_{i=1}^3 \frac{1}{(U^i - \bar{U}^i)^2} |F^{U^i}|^2, \\ (m_{\bar{i}\bar{i}}^{33})^2 &= \kappa_4^2 \left[\kappa_4^2 \widehat{V} G_{C_i^3 \bar{C}_i^3} - |F^{U^i}|^2 R_{U^i \bar{U}^i \bar{i}\bar{i}}^3 \right], \\ (m_{\bar{i}\bar{i}}^{7,j})^2 &= \kappa_4^2 \left[\kappa_4^2 \widehat{V} G_{C_i^{7,j} \bar{C}_i^{7,j}} - \sum_{k,l} F^{U^k} \bar{F}^{\bar{U}^l} R_{U^k \bar{U}^l \bar{i}\bar{i}}^{7,j} \right], \\ (m^{37_a})^2 &= \kappa_4^2 \left[\kappa_4^2 \widehat{V} G_{C^{37_a} \bar{C}^{37_a}} - \sum_{k,l} F^{U^k} \bar{F}^{\bar{U}^l} R_{U^k \bar{U}^l}^{37_a} \right], \\ (m^{7a7b})^2 &= \kappa_4^2 \left[\kappa_4^2 \widehat{V} G_{C^{7a7b} \bar{C}^{7a7b}} - \sum_{k,l} F^{U^k} \bar{F}^{\bar{U}^l} R_{U^k \bar{U}^l}^{7a7b} \right], \\ A_{IJK} &= i \prod_M (M - \bar{M})^{-1} \frac{\kappa_4^2 \lambda}{(2\pi)^2 \alpha'} \sum_i \int \bar{G}_3 \wedge \bar{\omega}_{A_i} (Y_{IJK} - (U^i - \bar{U}^i) \partial_{U^i} Y_{IJK}) \\ &\quad - i \prod_M (M - \bar{M})^{-1/2} \sum_{i=1}^3 F^{U^i} G^{\bar{C}_I C_I} \partial_{U^i} G_{C_I} (\bar{C}_I Y_{JK})_I, \end{aligned} \tag{4.20}$$

which is again zero for the case of I, J, K referring to C_i^3 . We will now substitute our solution (2.48) as an example. Here, we have

$$\begin{aligned}\frac{1}{(2\pi)^2\alpha'} \int \bar{G}_3 \wedge \omega_{A1} &= -4 (d_2 + d_3 + ib_2 + ib_3), \\ \frac{1}{(2\pi)^2\alpha'} \int \bar{G}_3 \wedge \omega_{A2} &= 4 (-d_1 - d_3 + ib_2 + id_0), \\ \frac{1}{(2\pi)^2\alpha'} \int \bar{G}_3 \wedge \omega_{A3} &= 4 (-d_1 - d_2 + ib_3 + id_0).\end{aligned}\tag{4.21}$$

For the F -terms, we get the following expressions:

$$\begin{aligned}F^{U^1} &= 2i\lambda\kappa_4^2 \prod (T^i - \bar{T}^i)^{-1/2} (-d_2 - d_3 + ib_2 + ib_3), \\ F^{U^2} &= -2i\lambda\kappa_4^2 \prod (T^i - \bar{T}^i)^{-1/2} (d_1 + d_2 + ib_2 + id_0), \\ F^{U^3} &= -2i\lambda\kappa_4^2 \prod (T^i - \bar{T}^i)^{-1/2} (d_1 + d_2 + ib_3 + id_0).\end{aligned}\tag{4.22}$$

So together with what we learned from eq. (4.6), \hat{V} is

$$\hat{V} = 4\lambda^2\kappa_4^2 \prod_i |T^i - \bar{T}^i|^{-1} \sum |B^i|^2.$$

(iv) $(3, 0)$ -flux

In this case, the only non-vanishing F -term is F^S , this corresponds to the “dilaton domination” SUSY breaking. The scalar potential in the bulk is

$$\hat{V} = -\frac{\kappa_4^{-2}}{(S - \bar{S})^2} |F^S|^2.$$

The gravitino mass $m_{3/2}$ is zero,

$$\begin{aligned}(m_{ii}^{33})^2 &= \kappa_4^4 \hat{V} G_{C_i^3 \bar{C}_i^3}, \\ (m_{ii}^{7,j})^2 &= \kappa_4^2 [\kappa_4^2 \hat{V} G_{C_i^7, j \bar{C}_i^{7,j}} - |F^S|^2 R_{S \bar{S} ii}^{7,j}], \\ (m^{37_a})^2 &= \kappa_4^2 [\kappa_4^2 \hat{V} G_{C^{37_a} \bar{C}^{37_a}} - |F^S|^2 R_{S \bar{S}}^{37_a}], \\ (m^{7a7b})^2 &= \kappa_4^2 [\kappa_4^2 \hat{V} G_{C^{7a7b} \bar{C}^{7a7b}} - |F^S|^2 R_{S \bar{S}}^{7a7b}], \\ A_{IJK} &= i Y_{IJK} \prod_M (M - \bar{M})^{-1} \frac{\kappa_4^2 \lambda}{(2\pi)^2 \alpha'} \int G_3 \wedge \bar{\Omega} \\ &\quad - i \prod_M (M - \bar{M})^{-1/2} F^S G^{\bar{C}_I C_I} \partial_S G_{C_I (\bar{C}_I Y_{JK}) I}, \\ m_{g, D7, j} &= F^S \frac{-\alpha'^2 f^k f^l}{(T^j - \bar{T}^j) - \alpha'^2 f^k f^l (S - \bar{S})}, \\ m_{g, D3} &= F^S \frac{1}{(S - \bar{S})} = -i \prod_M (M - \bar{M})^{-1/2} \frac{\lambda \kappa_4^2}{(2\pi)^2 \alpha'} \int G_3 \wedge \bar{\Omega}.\end{aligned}\tag{4.23}$$

We will now substitute our solution (2.51) as an example. Here, we have $\frac{1}{(2\pi)^2\alpha'} \int \overline{G}_3 \wedge \Omega = 8(d_3 - id_0)$. With this, F^S becomes

$$F^S = 4i\lambda\kappa_4^2 (d_3 + id_0) \prod_i (T^i - \overline{T}^i)^{-1/2}.$$

So \widehat{V} is

$$\widehat{V} = 4\lambda^2\kappa_4^2 (d_3^2 + d_0^2) \prod_i |T^i - \overline{T}^i|^{-1},$$

and finally,

$$\begin{aligned} m_{g,D7j} &= 4i\lambda\kappa_4^2 (d_3 + id_0) \prod_i (T^i - \overline{T}^i)^{-1/2} \frac{-\alpha'^2 f^k f^l}{(T^j - \overline{T}^j) - 2i\alpha'^2 f^k f^l}, \\ m_{g,D3} &= 2\lambda\kappa_4^2 (d_3 + id_0) \prod_i (T^i - \overline{T}^i)^{-1/2}. \end{aligned} \tag{4.24}$$

5. F -theory description

The description so far is valid as long as we are in the regime of small string coupling constant $g_s \sim e^{1/2\phi_{10}}$ and string tension $\alpha' \sim M_{string}^{-2}$ (or large radius limit). As we have seen, flux-quantization usually fixes the dilaton at finite values $g_s \sim 1$. Let us emphasize, that in the orientifold construction introduced in section 2, the localized tadpoles of both the $O3$ - and $O7$ -planes are cancelled in a non-local way. This quite generically produces a non-constant dilaton. This means, that the supergravity equations of motion lead to a dependence of the dilaton τ over the internal compact manifold. The natural setup to describe compactifications with a dilaton varying over the compactification manifold is F -theory compactified on elliptically fibered fourfolds. In this section, we shall lift our type IIB orientifold construction to F -theory. This allows a more thorough discussion of the supersymmetry breaking effects discussed before. Moreover, it provides the non-perturbative formulation of the type IIB orientifold constructions with background fluxes.

We shall discuss F -theory compactified on a fourfold X_8 [43,44]. The latter is an elliptic fibration over a threefold base X_6 , to be specified later. This compactification gives rise to $N=1$ supersymmetry in $D=4$ and the low-energy properties are determined by the fourfold, the $D3$ - and $D7$ -brane configurations. The complex structure modulus of the fiber, being a non-trivial function on the base coordinates, represents the complex coupling constant. The fluxes $G_3 \in H^3(X_6, \mathbf{Z})$ of the type IIB superstring become elements of the integer cohomology $G_4 \in H^4(X_8, \mathbf{Z})$ of the manifold X_8 . In F -theory compactified on the fourfold X_8 with background fluxes G_4 , the tadpole condition (2.28) becomes [35]:

$$N_{D3} = \frac{1}{24} \chi(X_8) - \frac{1}{8\pi^2} \int_{X_8} G_4 \wedge G_4 - \sum_{a=1}^{N_{D7}} \int_{C_{a;4}} F \wedge F. \tag{5.1}$$

We have N_{D7} $D7$ -branes wrapped around 4-cycles of the base manifold X_6 . The last term accounts for the total number of instantons inside the $D7$ -branes, which arise from background gauge bundles on the compact part of the $D7$ -brane world-volume. The latter break the gauge group down from the maximal one, dictated by the singularity of the elliptic fibration. W.r.t. the base manifold X_6 , the singularity is a complex codimension 2 locus, around which the $D7$ -brane is wrapped. The term proportional to the Euler number $\chi(X_8)$ of the fourfold X_8 accounts for the total induced $D3$ -brane charge coming from wrapping the $D7$ -branes over 4-cycles of the base X_6 [45]. The Euler number $\chi(X_8)$ of the fourfold X_8 is given by:

$$\chi(X_8) = \int_{X_8} I_8(R) \quad , \quad I_8(R) = \frac{1}{192} \left[\text{tr} R^4 - \frac{1}{4} (\text{tr} R^2)^2 \right] . \quad (5.2)$$

The F -theory lift of the superpotential (2.32) becomes [23]:

$$\widehat{W} = \int_{X_8} \Omega_4 \wedge G_4 . \quad (5.3)$$

In the following, we shall consider the fourfold

$$X_8 = \frac{T^6/\Gamma \times T^2}{\mathbf{Z}_2} , \quad (5.4)$$

with $\Gamma = \mathbf{Z}_2 \times \mathbf{Z}_2$ being the group (2.2) introduced in section 2. A third \mathbf{Z}_2 generator accounts for the elliptic fibration over our three-fold base $X_6 = T^6/\Gamma$, discussed in the previous sections. More concretely, we have the three \mathbf{Z}_2 -actions

$$\begin{aligned} \theta_1 &: (z^1, z^2, z^3, z^4) \longrightarrow (-z^1, -z^2, z^3, z^4) , \\ \theta_2 &: (z^1, z^2, z^3, z^4) \longrightarrow (z^1, -z^2, -z^3, z^4) , \\ \theta_3 &: (z^1, z^2, z^3, z^4) \longrightarrow (z^1, z^2, -z^3, -z^4) \end{aligned} \quad (5.5)$$

acting on the four internal complex coordinates z^i , $i = 1, 2, 3, 4$. The two generators θ_1 and θ_2 are the generators (2.2). The third generator θ_3 reflects both the coordinates z^3 and z^4 . The latter is the complex coordinate of the elliptic fiber T^2 .

Let us briefly discuss the orientifold limit of this F -theory compactification, *i.e.* how our previously studied $\Omega(-1)^{FL} I_6$ -orientifold of $T^6/\mathbf{Z}_2 \times \mathbf{Z}_2$ arises. Reflecting the fiber coordinate $z^4 \rightarrow -z^4$ corresponds to the monodromy element $-\mathbf{1}_2 \in SL(2, \mathbf{Z})$, changing the sign of the two 2-form fields B_2 and C_2 . However, it leaves all other massless fields invariant. It represents a perturbative symmetry equivalent to the orientifold action $\Omega(-1)^{FL}$ [46]. Hence, the generator θ_3 corresponds to the orientifold group element

$\theta_3 \equiv \Omega(-1)^{F_L} I_2^3$, discussed in section 2. Furthermore, we have the following identifications: $\theta_2\theta_3 \equiv \Omega(-1)^{F_L} I_2^2$, $\theta_1\theta_2\theta_3 \equiv \Omega(-1)^{F_L} I_2^1$ and $\theta_1\theta_3 \equiv \Omega(-1)^{F_L} I_6$. The positions of the $D7$ -branes and $O7$ -planes on the base X_6 are given by the zeros of the discriminant of the elliptic fiber in an appropriate constant coupling limit.

The manifold $(T^2)^4/\mathbf{Z}_2^3$ belongs to the family of Borcea fourfolds. The latter are singular Calabi–Yau fourfolds with $SU(4)$ holonomy, defined by the quotient $(K_3 \times K_3)/\mathbf{Z}_2$. Here, the \mathbf{Z}_2 acts as an involution, described by (r_1, a_1, δ_1) on the first $K3$, and as an involution, described by (r_2, a_2, δ_2) on the second $K3$. It reverses the sign of the $(2, 0)$ -forms of each $K3$, but leaves the $(4, 0)$ -form of the fourfold X_8 invariant. Their Euler numbers are given by $\chi = 288 + 6(r_1 - 10)(r_2 - 10)$. The fourfold (5.4) under consideration corresponds¹³ to the case¹⁴, with $r_1 = r_2 = 18$, $a_1 = a_2 = 4$, and $\delta_1 = \delta_2 = 0$. Hence for the manifold (5.4) we have:

$$\chi(X_8) = 672 . \quad (5.6)$$

With this information, the tadpole condition (5.1) becomes:

$$N_{D3} = 28 - \frac{1}{8\pi^2} \int_{X_8} G_4 \wedge G_4 - \sum_{a=1}^{N_{D7}} \int_{C_{a;4}} F \wedge F . \quad (5.7)$$

Without 4-form fluxes, *i.e.* $G_4 = 0$, this equation reduces to:

$$N_{D3} = 28 - \sum_{a=1}^{N_{D7}} \int_{C_{a;4}} F \wedge F . \quad (5.8)$$

This equation essentially describes the tadpole condition (2.12) of the type *IIB* orientifold discussed in the previous sections. Without instanton bundles on the $D7$ -branes we would conclude, that 28 $D3$ -branes are necessary to cancel the Ramond 4-form charge. However, some care is necessary for the interpretation of *Eq.* (5.8) [47]: 24 of the 28 $D3$ -branes are dissolved into instantons on the 96 $D7$ -branes, leaving us with 4 $D3$ -branes. The latter are placed on the orientifold $O3$ -planes and each $D3$ -brane appears in an orbit of 8 due to the group elements θ_1, θ_2 and Ω . Hence effectively, we have $4 \times 8 = 32$ $D3$ -branes in agreement with the model presented after *Eq.* (2.21).

¹³ Other choices would correspond to type *IIB* orientifolds with discrete torsion.

¹⁴ Note, that the orbifold $X_6 = T^6/(\mathbf{Z}_2 \times \mathbf{Z}_2)$ itself corresponds to an elliptic fibration of Voison–Borcea type $(K_3 \times T^2)/\mathbf{Z}_2$, with the \mathbf{Z}_2 -action described by the element θ_2 . The latter reverses the sign of the $(1, 0)$ -form of the T^2 and the sign of the $(2, 0)$ -form of the K_3 . The numbers $(r_2, a_2, \delta_2) = (18, 4, 0)$ encode the *CY* data $h_{(1,1)}(X_6) = 51$ and $h_{(2,1)}(X_6) = 3$, provided the $K3$ is realized as T^4/\mathbf{Z}_2 orbifold limit.

A general 4-flux G_4 on X_8 may be expressed as the sum $G_4 = G_{(4,0)} + G_{(3,1)} + G_{(2,2)} + G_{(1,3)} + G_{(0,4)}$, written as a linear combination of elements of $H^{(p,q)}(X_8, \mathbf{C})$, with $p+q = 4$. Since $h_{(4,0)}(X_8) = h_{(0,4)}(X_8) = 1$, we have the components:

$$dz^1 \wedge dz^2 \wedge dz^3 \wedge dz^4 \quad , \quad d\bar{z}^1 \wedge d\bar{z}^2 \wedge d\bar{z}^3 \wedge d\bar{z}^4 . \quad (5.9)$$

Besides, due to $h_{(3,1)}(X_8) = 4$, we have the $(3,1)$ -components:

$$\begin{aligned} d\bar{z}^1 \wedge dz^2 \wedge dz^3 \wedge dz^4 \quad , \quad dz^1 \wedge d\bar{z}^2 \wedge dz^3 \wedge dz^4 \\ dz^1 \wedge dz^2 \wedge d\bar{z}^3 \wedge dz^4 \quad , \quad dz^1 \wedge dz^2 \wedge dz^3 \wedge d\bar{z}^4 . \end{aligned} \quad (5.10)$$

Similarly, for the $(1,3)$ -components, with $h_{(1,3)}(X_8) = 4$. Finally, due to $h_{(2,2)}(X_8) = 460$, we have many $(2,2)$ components from the untwisted and twisted sector. The untwisted sector gives rise to the six $(2,2)$ -components:

$$\begin{aligned} dz^1 \wedge dz^2 \wedge d\bar{z}^3 \wedge d\bar{z}^4 \quad , \quad dz^1 \wedge d\bar{z}^2 \wedge dz^3 \wedge d\bar{z}^4 \quad , \quad d\bar{z}^1 \wedge dz^2 \wedge dz^3 \wedge d\bar{z}^4 \\ dz^1 \wedge d\bar{z}^2 \wedge d\bar{z}^3 \wedge dz^4 \quad , \quad d\bar{z}^1 \wedge dz^2 \wedge d\bar{z}^3 \wedge dz^4 \quad , \quad d\bar{z}^1 \wedge d\bar{z}^2 \wedge dz^3 \wedge dz^4 . \end{aligned} \quad (5.11)$$

In the following, we shall not discuss the remaining 454 $(2,2)$ -forms corresponding to flux components from the twisted sector. It is assumed that their quantization rules freeze the blowing up moduli of the orbifold (5.4). All the fluxes $G_{(p,q)}$, displayed above, are invariant under the orbifold group (5.5). In addition, they are primitive¹⁵, *i.e.* they fulfill $J^{5-p-q} \wedge G_{(p,q)} = 0$, with the Kähler form J on X_8 : $J = i \sum_{i=1}^4 \mathcal{T}_2^i dz^i \wedge d\bar{z}^i$. Here, \mathcal{T}_2^4 is the Kähler modulus of the fiber torus. Due to $\star F_{(p,4-p)} = (-1)^p F_{(p,4-p)}$, which holds for primitive flux components [23], the fluxes (5.10) are anti-selfdual. The remaining fluxes (5.9) and (5.11) are self-dual. In the case of unbroken supersymmetry, the 4-form fluxes G_4 have to be self-dual elements $G_{(2,2)}$ [48], leading to a vanishing superpotential (5.3). On the other hand, the components $G_{(4,0)}$ and $G_{(0,4)}$ break supersymmetry with a vanishing scalar potential. Only anti-selfdual components of G_4 , *i.e.* $G_{(3,1)}$ and $G_{(1,3)}$, lead to a non-vanishing scalar potential [23,49]. To this end, in our case, a non-vanishing scalar potential arises only from non-vanishing components $G_{(3,1)}$ and $G_{(1,3)}$.

The supersymmetry preserving $(2,1)$ -form fluxes G_3 , found in subsection 2.2 for our type *IIB* orientifold compactification on X_6 , may be directly lifted to self-dual $(2,2)$ -form fluxes G_4 of F -theory. Lorentz-invariance in $D = 4$ demands, that one component of the four-flux G_4 refers to the elliptic fiber. The other three components refer to the base manifold X_6 [35]. Another argument is, that a self-dual integer 4-form flux may describe

¹⁵ At any rate, on a fourfold, all components are primitive, except $G_{(2,2)}$. The latter may have primitive components being self-dual and non-primitive ones being anti-selfdual.

an F -theory limit only if it has one of its components w.r.t. the fiber [23]. Hence for that case, the 4-form flux may be written in the $SL(2, \mathbf{Z})_S$ -invariant combination:

$$G_4 = \frac{2\pi}{(S - \overline{S})} (G_3 \wedge d\overline{z}^4 - \overline{G}_3 \wedge dz^4) . \quad (5.12)$$

This form of the 4-flux is at least appropriate in a local trivialization of the elliptic fibration [12].

In the following, let us make contact to our findings from subsection 2.3. When we insert $dz^4 = dx^4 + Sdy^4$ into the above equation, we find

$$G_4 = -2\pi [F_3 \wedge dy^4 - H_3 \wedge dx^4] . \quad (5.13)$$

We now want to express G_4 through the language of section 2.3. To do so, we first define our basis for the 4-form flux:

$$\begin{aligned} \tilde{\alpha}_0 &= \alpha_0 \wedge dx^4, & \tilde{\alpha}_i &= \alpha_i \wedge dy^4, \quad i = 1, \dots, 3, \\ \tilde{\gamma}^0 &= \beta_0 \wedge dy^4, & \tilde{\gamma}^i &= \beta_i \wedge dx^4, \quad i = 1, \dots, 3, \\ \tilde{\beta}_0 &= \alpha_0 \wedge dy^4, & \tilde{\beta}_i &= \alpha_i \wedge dx^4, \quad i = 1, \dots, 3 \\ \tilde{\delta}^0 &= \beta_0 \wedge dx^4, & \tilde{\delta}^i &= \beta_i \wedge dy^4, \quad i = 1, \dots, 3, \end{aligned} \quad (5.14)$$

where the α , γ correspond to ISD fluxes and the β , δ to IASD fluxes. Expressed in this basis, our lifted 3-form flux becomes

$$\begin{aligned} \frac{-1}{(2\pi)^3 \alpha'} G_4 &= c^0 \tilde{\alpha}_0 + \sum_{i=1}^3 a^i \tilde{\alpha}_i + \sum_{i=1}^3 d_i \tilde{\gamma}^i + b_0 i \tilde{\gamma}^0 \\ &+ a^0 \tilde{\beta}_0 + \sum_{i=1}^3 c^i \tilde{\beta}_i + \sum_{i=1}^3 b_i \tilde{\delta}^i + d_0 \tilde{\delta}^0, \end{aligned} \quad (5.15)$$

with the a , b , c , d the coefficients of our original 3-form flux. Now we want to do the same in complex notation. We define the following complex basis:

$$\begin{aligned} \tilde{\omega}_{A0} &= \omega_{B0} \wedge dz^4, & \tilde{\omega}_{Ai} &= \omega_{Bi} \wedge dz^4, \\ \tilde{\omega}_{B0} &= \omega_{B0} \wedge d\overline{z}^4, & \tilde{\omega}_{Bi} &= \omega_{Ai} \wedge d\overline{z}^4, \\ \tilde{\omega}_{C0} &= \omega_{A0} \wedge d\overline{z}^4, & \tilde{\omega}_{Ci} &= \omega_{Ai} \wedge dz^4, \\ \tilde{\omega}_{D0} &= \omega_{A0} \wedge dz^4, & \tilde{\omega}_{Di} &= \omega_{Bi} \wedge d\overline{z}^4, \end{aligned} \quad (5.16)$$

where the ω_A , ω_C correspond to the ISD fluxes and the ω_B , ω_D correspond to the IASD fluxes, and $\overline{\tilde{\omega}_A} = \tilde{\omega}_C$, $\overline{\tilde{\omega}_B} = \tilde{\omega}_D$. Expressed through the $\tilde{\alpha}$, $\tilde{\beta}$, $\tilde{\gamma}$, $\tilde{\delta}$ and the U^i and S , $\tilde{\omega}_{A0}$ for example has the following form:

$$\tilde{\omega}_{A0} = \tilde{\alpha}_0 + S\tilde{\beta}_0 + \sum_{i=1}^3 \overline{U}^i (\tilde{\beta}_i + S\tilde{\alpha}_i) - \sum_{i \neq j \neq k} \overline{U}^j \overline{U}^k (\tilde{\gamma}^i + S\tilde{\delta}^i) + \overline{U}^1 \overline{U}^2 \overline{U}^3 (\tilde{\delta}^0 + S\tilde{\gamma}^0). \quad (5.17)$$

The form of the other ω can be easily deduced from the above and (2.36). This basis fulfills

$$\int \omega_{Ai} \wedge \omega_{Ci} = - \int \omega_{Bi} \wedge \omega_{Di} = - \prod_{j=1}^3 (U^j - \bar{U}^j)(S - \bar{S}), \quad i = 0, \dots, 3 \quad (5.18)$$

while all other combinations of the basis elements are zero. Expressed through this complex basis, our 4-form flux (5.12) has the following form:

$$\begin{aligned} \frac{(S - \bar{S})}{(2\pi)^3 \alpha'} G_4 = & A^0 \tilde{\omega}_{C0} + \sum_{i=1}^3 A^i \tilde{\omega}_{Ci} + B^0 \tilde{\omega}_{B0} + \sum_{i=1}^3 B^i \tilde{\omega}_{Di} \\ & - \bar{A}^0 \tilde{\omega}_{A0} - \sum_{i=1}^3 \bar{A}^i \tilde{\omega}_{Ai} - \bar{B}^0 \tilde{\omega}_{D0} - \sum_{i=1}^3 \bar{B}^i \tilde{\omega}_{Bi} , \end{aligned} \quad (5.19)$$

where again the A^i , B^i are the complex coefficients of our original 3-form flux. With this, we find:

$$N_{flux} = -\frac{1}{8\pi^2} \int G_4 \wedge G_4 = - \frac{\prod_{i=1}^3 (U^i - \bar{U}^i)}{(S - \bar{S})} \left(\sum_{i=0}^3 |A^i|^2 - \sum_{i=0}^3 |B^i|^2 \right) , \quad (5.20)$$

which agrees with N_{flux} from Eq. (2.39). The anomaly equation (5.7) becomes:

$$N_{D3} = 28 + N_{flux} - \sum_{a=1}^{N_{D7}} \int_{C_{a;4}} F \wedge F . \quad (5.21)$$

So far, we have described the F -theory lift of our type IIB orientifold construction with 3-form fluxes. Generically, F -theory compactified on Calabi–Yau fourfolds with G_4 -fluxes leads to a warped metric. See Refs. [23,35,50,51,52] for an account on this subject. The next step would be to work out the differential equation for the warp factor following from the equations of motion for the 4-form field C_4 . Furthermore, from the equation of motion for the internal metric and dilaton field, a system of differential equations follows for the (non-constant) dilaton field. We reserve this for future work.

6. Concluding remarks

In this article, we have determined the $D = 4, N = 1$ tree-level action for a class of type IIB orientifold models with $D3$ - and $D7$ -branes up to second order in the matter fields. These models are based on toroidal orbifold/orientifold compactifications of type IIB . The action, summarized in the three functions K, W and f , depends on both closed and open string moduli fields. Generically, the closed string moduli fields describe the

geometry of the underlying type *IIB* compactification and the open string moduli account for moduli fields originating from the *D*-branes like matter fields, Wilson lines and the *D*-brane positions. We have calculated the Kähler metrics of these moduli fields extending the results of *Ref.* [1]. In particular, in subsection 3.3. the metric for matter fields originating from a 1/2 BPS system of two *D*-branes is elaborated upon. We have allowed for both non-vanishing *RR* and *NSNS* 3-form fluxes in the bulk and 2-form fluxes (instantons) on the *D7*-brane world-volume. While the former allow for fixing the dilaton and complex structure moduli due to their internal flux quantization conditions, non-vanishing 2-form fluxes give rise to a *D*-term potential fixing a part of the Kähler moduli. The Kähler potential *K* and the gauge kinetic function *f* only depend non-trivially on the 2-form fluxes (*cf. Eqs.* (3.10) and (3.34)). On the other hand, as it is well-known, the 3-form fluxes enter the holomorphic superpotential *W* (*cf.* (2.32)). The latter depends on the dilaton and complex structure moduli only. A fact, which is at least true in the no-scale case. However, since the calculation of the scalar potential and possible soft-supersymmetry breaking terms involves both the Kähler potential, gauge kinetic function and the superpotential, those terms depend non-trivially both on 2- and 3-form fluxes (*cf. Eqs.* (4.15) and (4.16)).

Generically, most of the discussions on superstring vacua with background fluxes is at the level of the lowest order expansion in the string coupling g_s and string tension α' of the underlying superstring effective action. Hence the stabilization of some moduli takes place at string tree-level. In contrast, one should mention, that there exist purely *stringy* constructions like *M*-theory *U*-duality orbifolds with a large number of moduli frozen by the orbifold group and with very few moduli left unfixed [53]. In type *IIB*, the superpotential is exact to all orders in α' as it only depends on the dilaton and complex structure moduli and does not receive world-sheet instanton corrections. However, through α' -corrections to the Kähler potential, eventually also the *F*-terms and the scalar potential receive corrections. Hence, in general, these effects have to be taken into account and it is certainly desirable to go beyond the lowest order approximation. Already if one includes one-loop effects into the Kähler potential, the no-scale structure is generically lost [54]. In addition, the no-go theorem of supergravity, that only a certain class of fluxes (namely ISD-fluxes) lead to a consistent supergravity solution may be disproved by some additional stringy effects. Hence the study of effects, which go beyond the supergravity approximation, is very important.

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Appendix A. Fluxes in the presence of $D3$ and $D7$ -branes

One way one can think of turning on flux on a Dp -brane is via the generalized Scherk-Schwarz Ansatz:

$$B_{mn} = H_{mnp}x^p. \quad (\text{A.1})$$

p may run only over the coordinates transversal to the brane, so in the case of a $D7$ -brane, which fills the directions x_0, \dots, x_7 (wrapping the tori $T_1^2 \times T_2^2$), we have

$$B_{mn}^7 = H_{mn8}x^8 + H_{mn9}x^9. \quad (\text{A.2})$$

As H_{ijk} must always have one index equal to either 8 or 9, not all of the 20 possible components are allowed in our case. Not allowed are the fluxes

$$\begin{aligned} dx^1 \wedge dy^1 \wedge dx^2 & & dx^1 \wedge dy^1 \wedge dy^2 \\ dx^1 \wedge dx^2 \wedge dy^2 & & dy^1 \wedge dx^2 \wedge dy^2. \end{aligned} \quad (\text{A.3})$$

Expressed in complex notation, this would correspond to the fluxes $H_{1\bar{1}2}$, $H_{1\bar{2}2}$, $H_{1\bar{1}\bar{2}}$, $H_{2\bar{1}\bar{2}}$.

For $D7$ -branes wrapping the tori $T_1^2 \times T_3^2$ or $T_2^2 \times T_3^2$, we find similar results. Having a setup of three stacks of $D7$ branes, one stack not wrapping T_3^2 , one not wrapping T_2^2 , and one not wrapping T_1^2 , we lose 12 of the twenty flux components and are left with fluxes, which have one index on each of the tori.

Appendix B. Components of the curvature tensor

We will first derive the components coming from the metric for the matter fields on the 3-branes. Due to the diagonal structure of the metric, many components of the curvature tensor are zero. We are thus left only with the following non-zero components:

$$\begin{aligned} R_{T^i \bar{T}^i i \bar{i}}^3 &= \frac{-1}{(T^i - \bar{T}^i)^2} G_{C_i^3 \bar{C}_i^3}, \\ R_{U^i \bar{U}^i i \bar{i}}^3 &= \frac{-1}{(U^i - \bar{U}^i)^2} G_{C_i^3 \bar{C}_i^3}. \end{aligned} \quad (\text{B.1})$$

Now we will examine the components coming from the metric for matter fields on the stack of 7-branes not wrapping the j 'th torus. If we assume the 2-form flux to be zero, we arrive at the same component structure as for the 3-brane. Turning on the 2-form flux makes life a lot harder, as the $\tilde{f}^i = \frac{f^i}{\text{Im} T^i}$ depend on all of the T^i and on S . Only few components are zero now. The only one that retains the simple structure from above is

$$R_{U^i U^i i \bar{i}}^{7,j} = \frac{-1}{(U^i - \bar{U}^i)^2} G_{C_i^{7,j} \bar{C}_i^{7,j}}. \quad (\text{B.2})$$

For the other components, we need to know the $\partial_M(\text{Im}\mathcal{T}^j)$, where M runs over S, T^i . They can be obtained by taking the total derivative of the imaginary part of equations (2.6) and (2.7) and inverting the partial derivatives. We find

$$\begin{aligned}\frac{\partial(\text{Im}\mathcal{T}^j)}{\partial S} &= \frac{-i}{4} \frac{\text{Im}\mathcal{T}^j}{\text{Im}S}, \\ \frac{\partial(\text{Im}\mathcal{T}^j)}{\partial T^j} &= \frac{-i}{4} \frac{\text{Im}\mathcal{T}^j}{\text{Im}T^j}, \\ \frac{\partial(\text{Im}\mathcal{T}^j)}{\partial T^k} &= \frac{i}{4} \frac{\text{Im}\mathcal{T}^j}{\text{Im}T^k}, \quad j \neq k.\end{aligned}\tag{B.3}$$

We will also need the $\partial_M \partial_{\overline{N}}(\text{Im}\mathcal{T}^j)$:

$$\begin{aligned}\frac{\partial(\text{Im}\mathcal{T}^j)}{\partial S \partial \overline{S}} &= \frac{-3}{16} \frac{(\text{Im}\mathcal{T}^j)}{(\text{Im}S)^2}, & \frac{\partial(\text{Im}\mathcal{T}^j)}{\partial T^j \partial \overline{S}} &= \frac{-1}{16} \frac{(\text{Im}\mathcal{T}^j)}{(\text{Im}S)(\text{Im}T^j)}, \\ \frac{\partial(\text{Im}\mathcal{T}^j)}{\partial T^k \partial \overline{S}} &= \frac{1}{16} \frac{(\text{Im}\mathcal{T}^j)}{(\text{Im}S)(\text{Im}T^k)}, & \frac{\partial(\text{Im}\mathcal{T}^j)}{\partial T^k \partial \overline{T}^i} &= \frac{-1}{16} \frac{(\text{Im}\mathcal{T}^j)}{(\text{Im}T^i)(\text{Im}T^k)}, \\ \frac{\partial(\text{Im}\mathcal{T}^j)}{\partial T^k \partial \overline{T}^j} &= \frac{1}{16} \frac{(\text{Im}\mathcal{T}^j)}{(\text{Im}T^j)(\text{Im}T^k)}, & \frac{\partial(\text{Im}\mathcal{T}^j)}{\partial T^k \partial \overline{T}^k} &= \frac{1}{16} \frac{(\text{Im}\mathcal{T}^j)}{(\text{Im}T^k)^2}, \\ \frac{\partial(\text{Im}\mathcal{T}^j)}{\partial T^j \partial \overline{T}^j} &= \frac{-3}{16} \frac{(\text{Im}\mathcal{T}^j)}{(\text{Im}T^j)^2},\end{aligned}\tag{B.4}$$

where M, N run over S, T^i . Now we can write down the remaining $R^{7,j}$:

$$\begin{aligned}R_{MNi\bar{i}}^{7,j} &= \frac{-\kappa_4^{-2}}{(T^k - \overline{T}^k)(U^i - \overline{U}^i)} \frac{1}{|1 + i\tilde{f}^i|^3 |1 + i\tilde{f}^k|} \times \\ &\times \left[|1 + i\tilde{f}^k|^2 \partial_M |1 + i\tilde{f}^i| \partial_{\overline{N}} |1 + i\tilde{f}^i| - \right. \\ &- |1 + i\tilde{f}^i| \{ |1 + i\tilde{f}^i| \partial_M |1 + i\tilde{f}^k| \partial_{\overline{N}} |1 + i\tilde{f}^k| + \\ &+ |1 + i\tilde{f}^k| (|1 + i\tilde{f}^k| \partial_M \partial_{\overline{N}} |1 + i\tilde{f}^i| - |1 + i\tilde{f}^i| \partial_M \partial_{\overline{N}} |1 + i\tilde{f}^k|) \} \left. \right], \quad (M, N) \neq (T^k, T^k), \\ R_{T^k \overline{T}^k i \bar{i}}^{7,j} &= \frac{-1}{(T^k - \overline{T}^k)^2} G_{C_i^{7,j} \overline{C}_i^{7,j}} - \frac{\kappa_4^{-2}}{(T^k - \overline{T}^k)(U^i - \overline{U}^i)} \frac{1}{|1 + i\tilde{f}^i|^3 |1 + i\tilde{f}^k|} \times \\ &\times \left[|1 + i\tilde{f}^k|^2 \partial_M |1 + i\tilde{f}^i| \partial_{\overline{N}} |1 + i\tilde{f}^i| - \right. \\ &- |1 + i\tilde{f}^i| \{ |1 + i\tilde{f}^i| \partial_M |1 + i\tilde{f}^k| \partial_{\overline{N}} |1 + i\tilde{f}^k| + \\ &+ |1 + i\tilde{f}^k| (|1 + i\tilde{f}^k| \partial_M \partial_{\overline{N}} |1 + i\tilde{f}^i| - |1 + i\tilde{f}^i| \partial_M \partial_{\overline{N}} |1 + i\tilde{f}^k|) \} \left. \right],\end{aligned}\tag{B.5}$$

$$\begin{aligned}
R_{M\bar{N}j\bar{j}}^{7,j} &= \frac{-\kappa_4^{-2}}{(S-\bar{S})(U^j-\bar{U}^j)} \left[\partial_M \partial_{\bar{N}} |1 - \tilde{f}^i \tilde{f}^k| - \partial_M |1 - \tilde{f}^i \tilde{f}^k| \partial_{\bar{N}} |1 - \tilde{f}^i \tilde{f}^k| \frac{1}{|1 - \tilde{f}^i \tilde{f}^k|} \right], \\
(M, N) &\neq (S, S), \\
R_{S\bar{S}j\bar{j}}^{7,j} &= \frac{-1}{(S-\bar{S})^2} G_{C_j^{7,j} \bar{C}_j^{7,j}} - \frac{\kappa_4^{-2}}{(S-\bar{S})(U^j-\bar{U}^j)} \left[\partial_M \partial_{\bar{N}} |1 - \tilde{f}^i \tilde{f}^k| \right. \\
&\quad \left. - \partial_M |1 - \tilde{f}^i \tilde{f}^k| \partial_{\bar{N}} |1 - \tilde{f}^i \tilde{f}^k| \frac{1}{|1 - \tilde{f}^i \tilde{f}^k|} \right], \\
R_{M\bar{U}^i \bar{i}}^{7,j} &= R_{M\bar{U}^j \bar{j}}^{7,j} = 0,
\end{aligned} \tag{B.6}$$

where

$$\begin{aligned}
\partial_M |1 + i\tilde{f}^k| &= -|1 + i\tilde{f}^k|^{-1} \frac{(f^k)^2}{(\text{Im}\mathcal{T}^k)^3} \partial_M (\text{Im}\mathcal{T}^k), \\
\partial_M \partial_{\bar{N}} |1 + i\tilde{f}^k| &= |1 + i\tilde{f}^k|^{-1} \frac{(f^k)^2}{(\text{Im}\mathcal{T}^k)^4 [(f^k)^2 + (\text{Im}\mathcal{T}^k)^2]} \{ [2(f^k)^2 + 3(\text{Im}\mathcal{T}^k)^2] \times \\
&\quad \times \partial_M (\text{Im}\mathcal{T}^k) \partial_{\bar{N}} (\text{Im}\mathcal{T}^k) - \text{Im}\mathcal{T}^k [(f^k)^2 + (\text{Im}\mathcal{T}^k)^2] \partial_M \partial_{\bar{N}} (\text{Im}\mathcal{T}^k) \}, \\
\partial_M |1 - \tilde{f}^i \tilde{f}^k| &= \frac{-f^i f^k}{(\text{Im}\mathcal{T}^i)^2 (\text{Im}\mathcal{T}^k)^2} [(\text{Im}\mathcal{T}^k) \partial_M (\text{Im}\mathcal{T}^i) + (\text{Im}\mathcal{T}^i) \partial_M (\text{Im}\mathcal{T}^k)], \\
\partial_M \partial_{\bar{N}} |1 - \tilde{f}^i \tilde{f}^k| &= \frac{-f^i f^k}{(\text{Im}\mathcal{T}^i)^3 (\text{Im}\mathcal{T}^k)^3} \times \\
&\quad \times [-2(\text{Im}\mathcal{T}^k)^2 \partial_M (\text{Im}\mathcal{T}^i) \partial_{\bar{N}} (\text{Im}\mathcal{T}^i) - (\text{Im}\mathcal{T}^k) (\text{Im}\mathcal{T}^i) \partial_{\bar{N}} (\text{Im}\mathcal{T}^i) \partial_M (\text{Im}\mathcal{T}^k) \\
&\quad - (\text{Im}\mathcal{T}^k) (\text{Im}\mathcal{T}^i) \partial_M (\text{Im}\mathcal{T}^i) \partial_{\bar{N}} (\text{Im}\mathcal{T}^k) - 2(\text{Im}\mathcal{T}^i)^2 \partial_M (\text{Im}\mathcal{T}^k) \partial_{\bar{N}} (\text{Im}\mathcal{T}^k) \\
&\quad + (\text{Im}\mathcal{T}^i) (\text{Im}\mathcal{T}^k)^2 \partial_M \partial_{\bar{N}} (\text{Im}\mathcal{T}^i) + (\text{Im}\mathcal{T}^i)^2 (\text{Im}\mathcal{T}^k) \partial_M \partial_{\bar{N}} (\text{Im}\mathcal{T}^k)] .
\end{aligned} \tag{B.7}$$

Now we come to the terms coming from the twisted matter metrics. From the metric for matter fields going from the 3-branes to the a 'th of the 7-brane stacks, we get the following components. Again, $R_{U^j \bar{U}^j \bar{i} i}^{37,a}$ has the simplest structure and again, $R_{U^j \bar{U}^k \bar{i} i}^{37,a}$, $k \neq j$ is zero.

$$\begin{aligned}
R_{U^j \bar{U}^j}^{37,a} &= \frac{-\theta^j}{(U^j - \bar{U}^j)^2} G_{C^{37,a} \bar{C}^{37,a}}, \quad j \neq a \\
R_{U^j \bar{U}^j}^{37,j} &= 0, \\
R_{M\bar{U}^j}^{37,a} &= \frac{\partial_M \theta^j}{(U^j - \bar{U}^j)} G_{C^{37,a} \bar{C}^{37,a}}, \quad a \neq j, \\
R_{M\bar{U}^j}^{37,j} &= 0, \\
R_{M\bar{N}}^{37,a} &= - \sum_{j \neq a} \ln(U^j - \bar{U}^j) \partial_M \partial_{\bar{N}} (\theta^j) G_{C^{37,a} \bar{C}^{37,a}},
\end{aligned} \tag{B.8}$$

where

$$\begin{aligned}
\partial_M \theta^j &= -\frac{1}{\pi} \frac{1}{(1 + (\tilde{f}^j)^2)} \frac{f^j}{(\text{Im} \mathcal{T}^j)^2} \partial_M (\text{Im} \mathcal{T}^j), \\
\partial_M \partial_{\overline{N}} (\theta^j) &= -\frac{f^j}{\pi [(f^j)^2 + (\text{Im} \mathcal{T}^j)^2]^2} [-2(\text{Im} \mathcal{T}^j) \partial_M (\text{Im} \mathcal{T}^j) \partial_{\overline{N}} (\text{Im} \mathcal{T}^j) + \\
&\quad + [(f^j)^2 + (\text{Im} \mathcal{T}^j)^2] \partial_M \partial_{\overline{N}} (\text{Im} \mathcal{T}^j)] .
\end{aligned} \tag{B.9}$$

From the metric for matter fields going from the a 'th 7-brane stack to the b 'th 7-brane stack, we get

$$\begin{aligned}
R_{U^k \overline{U}^k}^{7a,7b} &= \frac{-\theta_{ab}^k}{(U^k - \overline{U}^k)^2} G_{C^{7a7b} \overline{C}^{7a7b}} \\
R_{U^j \overline{U}^k}^{7a,7b} &= 0, \quad j \neq k, \\
R_{M \overline{U}^k}^{7a,7b} &= \frac{\partial_M (\theta_{ab}^k)}{(U^k - \overline{U}^k)} G_{C^{7a7b} \overline{C}^{7a7b}}, \\
R_{M \overline{N}}^{7a,7b} &= \sum_l \left\{ -\ln(U^l - \overline{U}^l) \partial_M \partial_{\overline{N}} (\theta_{ab}^l) \right. \\
&\quad + \frac{1}{2} [\partial_M \partial_{\overline{N}} (\theta_{ab}^l) [\psi_0(\theta_{ab}^l) + \psi_0(1 - \theta_{ab}^l)] + \\
&\quad \left. + \partial_M (\theta_{ab}^l) \partial_{\overline{N}} (\theta_{ab}^l) [\psi_1(\theta_{ab}^l) - \psi_1(1 - \theta_{ab}^l)] \right\} G_{C^{7a7b} \overline{C}^{7a7b}},
\end{aligned} \tag{B.10}$$

where ψ_n is the n 'th Polygamma function and

$$\begin{aligned}
\partial_M \theta_{ab}^j &= -\frac{1}{\pi} \left[\frac{1}{1 + (\tilde{f}_b^j)^2} \frac{f_b^j}{(\text{Im} \mathcal{T}^j)^2} \partial_M (\text{Im} \mathcal{T}^j) - \frac{1}{1 + (\tilde{f}_a^j)^2} \frac{f_a^j}{(\text{Im} \mathcal{T}^j)^2} \partial_M (\text{Im} \mathcal{T}^j) \right], \\
\partial_M \partial_{\overline{N}} (\theta_{ab}^j) &= -\frac{1}{\pi} \left\{ \frac{f_b^j}{[(f_b^j)^2 + (\text{Im} \mathcal{T}^j)^2]^2} [-2(\text{Im} \mathcal{T}^j) \partial_M (\text{Im} \mathcal{T}^j) \partial_{\overline{N}} (\text{Im} \mathcal{T}^j) + \right. \\
&\quad + [(f_b^j)^2 + (\text{Im} \mathcal{T}^j)^2] \partial_M \partial_{\overline{N}} (\text{Im} \mathcal{T}^j)] - \\
&\quad - \frac{f_a^j}{[(f_a^j)^2 + (\text{Im} \mathcal{T}^j)^2]^2} [-2(\text{Im} \mathcal{T}^j) \partial_M (\text{Im} \mathcal{T}^j) \partial_{\overline{N}} (\text{Im} \mathcal{T}^j) + \\
&\quad \left. + [(f_a^j)^2 + (\text{Im} \mathcal{T}^j)^2] \partial_M \partial_{\overline{N}} (\text{Im} \mathcal{T}^j)] \right\} .
\end{aligned} \tag{B.11}$$

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